Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4597910349

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **October/November 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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1 **(a)** By differentiating e^{-x^2}, find the Maclaurin's series for e^{-x^2} up to and including the term in x^2. [5]

(b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

2 The variables x and y are related by the differential equation

$$9\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+6\frac{\mathrm{d}y}{\mathrm{d}x}+y=3x^2+30x.$$

(a) Find the general solution for y in terms of x. [6]

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(b) State an approximate solution for large positive values of x. [1]

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3 **(a)** Show that the system of equations

$$\begin{aligned} x-2y-4z &= 1, \\ x-2y+kz &= 1, \\ -x+2y+2z &= 1, \end{aligned}$$

where k is a constant, does not have a unique solution. [2]

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(b) Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that $k=-2$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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(d) For the case where $k\neq-2$ and $k\neq-4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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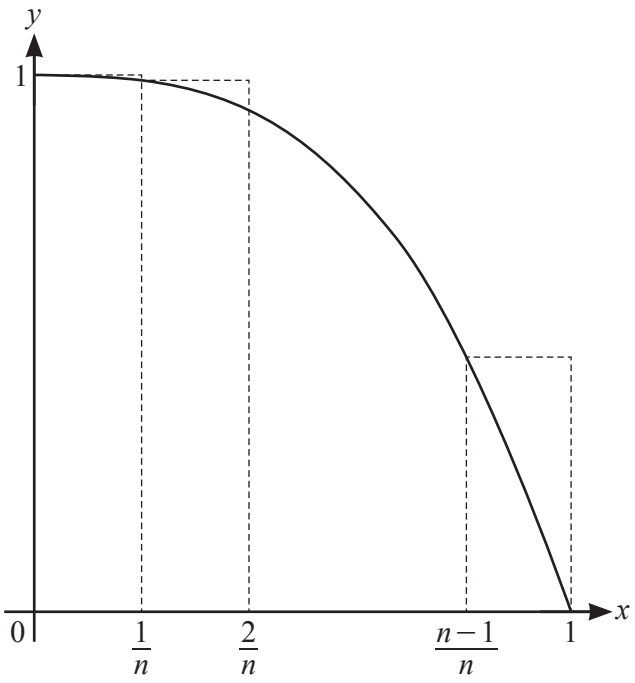
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The diagram shows the curve with equation $y = 1 - x^3$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1 - x^3)\,\mathrm{d}x \leqslant \frac{3n^2 + 2n - 1}{4n^2}. \qquad [4]$$

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(b) Use a similar method to find, in terms of n, a lower bound for $\int_0^1 (1-x^3)\,\mathrm{d}x$. [4]

5 It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t, show that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{1}{\sqrt{1-t^2}}$. [4]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer. [5]

6 **(a)** Use de Moivre's theorem to show that $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$. [5]

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(b) Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}\theta} + y\cot\theta = \sin^3\theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$. [6]

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

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(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

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The 3×3 matrix **A** has distinct eigenvalues b, -1, 1 with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

(c) Find **A** in terms of b. [4]

8 **(a)** Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

(b) Starting from the definitions of coth and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$

The curve C has equation $y = \ln\coth\left(\frac{1}{2}x\right)$ for $x > 0$.

(c) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosech} x$. [3]

(d) It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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