Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **October/November 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 8 2 4 8 7 7 1 9 0 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

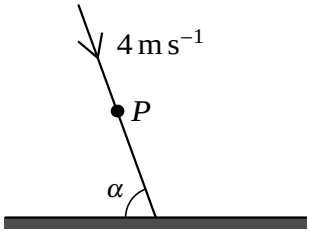
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

© UCLES 2014

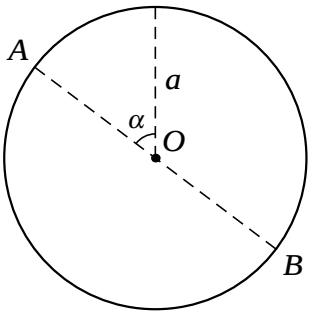
1 Two smooth spheres $\boldsymbol{A}$ and $\boldsymbol{B}$, of equal radii and masses $2\boldsymbol{m}$ and $\boldsymbol{m}$ respectively, lie at rest on a smooth horizontal table. The spheres $\boldsymbol{A}$ and $\boldsymbol{B}$ are projected directly towards each other with speeds $4\boldsymbol{u}$ and $3\boldsymbol{u}$ respectively. The coefficient of restitution between the spheres is $\boldsymbol{e}$. Find the set of values of $\boldsymbol{e}$ for which the direction of motion of $\boldsymbol{A}$ is reversed in the collision. [5]

2



A small smooth ball $\boldsymbol{P}$ is moving on a smooth horizontal plane with speed $4\,\text{m}\,\text{s}^{-1}$. It strikes a smooth vertical barrier at an angle α (see diagram). The coefficient of restitution between $\boldsymbol{P}$ and the barrier is 0.4. Given that the speed of $\boldsymbol{P}$ is halved as a result of the collision, find the value of α. [5]

3

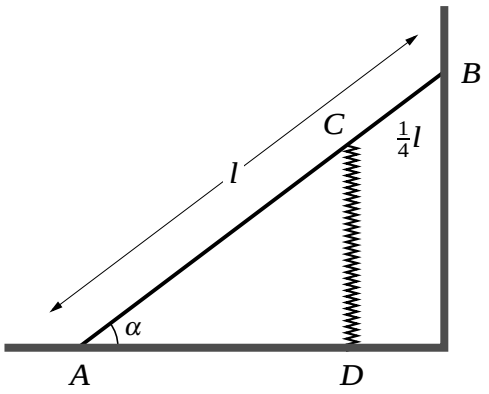


A smooth cylinder of radius $\boldsymbol{a}$ is fixed with its axis horizontal. The point $\boldsymbol{O}$ is the centre of a circular cross-section of the cylinder. The line $\boldsymbol{AOB}$ is a diameter of this circular cross-section and the radius $\boldsymbol{OA}$ makes an angle α with the upward vertical (see diagram). It is given that $\cos\alpha = \frac{3}{5}$. A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ moves on the inner surface of the cylinder in the plane of the cross-section. The particle passes through $\boldsymbol{A}$ with speed $\boldsymbol{u}$ along the surface in the downwards direction. The magnitude of the reaction between $\boldsymbol{P}$ and the inner surface of the sphere is $\boldsymbol{R_A}$ when $\boldsymbol{P}$ is at $\boldsymbol{A}$, and is $\boldsymbol{R_B}$ when $\boldsymbol{P}$ is at $\boldsymbol{B}$. It is given that $\boldsymbol{R_B} = 10\boldsymbol{R_A}$. Show that $\boldsymbol{u}^2 = \boldsymbol{ag}$. [6]

The particle loses contact with the surface of the cylinder when $\boldsymbol{OP}$ makes an angle θ with the upward vertical. Find the value of $\cos\theta$. [4]

© UCLES 2014

4



A uniform rod $\boldsymbol{AB}$, of length $\boldsymbol{l}$ and mass $\boldsymbol{m}$, rests in equilibrium with its lower end $\boldsymbol{A}$ on a rough horizontal floor and the end $\boldsymbol{B}$ against a smooth vertical wall. The rod is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$, and is in a vertical plane perpendicular to the wall. The rod is supported by a light spring $\boldsymbol{CD}$ which is in compression in a vertical line with its lower end $\boldsymbol{D}$ fixed on the floor. The upper end $\boldsymbol{C}$ is attached to the rod at a distance $\frac{1}{4}\boldsymbol{l}$ from $\boldsymbol{B}$ (see diagram). The coefficient of friction at $\boldsymbol{A}$ between the rod and the floor is $\frac{1}{3}$ and the system is in limiting equilibrium.

(i) Show that the normal reaction of the floor at $\boldsymbol{A}$ has magnitude $\frac{1}{2}\boldsymbol{mg}$ and find the force in the spring. [7]

(ii) Given that the modulus of elasticity of the spring is $2\boldsymbol{mg}$, find the natural length of the spring. [4]

5 The points $\boldsymbol{A}$ and $\boldsymbol{B}$ are on a smooth horizontal table at a distance $8\boldsymbol{a}$ apart. A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ lies on the table on the line $\boldsymbol{AB}$, between $\boldsymbol{A}$ and $\boldsymbol{B}$. The particle is attached to $\boldsymbol{A}$ by a light elastic string of natural length $3\boldsymbol{a}$ and modulus of elasticity $6\boldsymbol{mg}$, and to $\boldsymbol{B}$ by a light elastic string of natural length $2\boldsymbol{a}$ and modulus of elasticity $\boldsymbol{mg}$. In equilibrium, $\boldsymbol{P}$ is at the point $\boldsymbol{O}$ on $\boldsymbol{AB}$.

(i) Show that $\boldsymbol{AO} = 3.6\boldsymbol{a}$. [4]

The particle is released from rest at the point $\boldsymbol{C}$ on $\boldsymbol{AB}$, between $\boldsymbol{A}$ and $\boldsymbol{B}$, where $\boldsymbol{AC} = 3.4\boldsymbol{a}$.

(ii) Show that $\boldsymbol{P}$ moves in simple harmonic motion and state the period. [6]

(iii) Find the greatest speed of $\boldsymbol{P}$. [2]

6 A random sample of 50 observations of a random variable $\boldsymbol{X}$ and a random sample of 60 observations of a random variable $\boldsymbol{Y}$ are taken. The results for the sample means, $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$, and the unbiased estimates for the population variances, $\boldsymbol{s}_{\boldsymbol{x}}^2$ and $\boldsymbol{s}_{\boldsymbol{y}}^2$, respectively, are as follows.

$$\bar{\boldsymbol{x}} = 25.4 \qquad \bar{\boldsymbol{y}} = 23.6 \qquad \boldsymbol{s}_{\boldsymbol{x}}^2 = 23.2 \qquad \boldsymbol{s}_{\boldsymbol{y}}^2 = 27.8$$

A test, at the $\alpha\%$ significance level, of the null hypothesis that the population means of $\boldsymbol{X}$ and $\boldsymbol{Y}$ are equal against the alternative hypothesis that they are not equal is carried out. Given that the null hypothesis is not rejected, find the set of possible values of α. [5]

© UCLES 2014

7 The time, T seconds, between successive cars passing a particular checkpoint on a wide road has probability density function f given by

$$f(t) = \begin{cases} \frac{1}{100}e^{-0.01t} & t \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) State the expected value of T. [1]

(ii) Find the median value of T. [3]

Sally wishes to cross the road at this checkpoint and she needs 20 seconds to complete the crossing. She decides to start out immediately after a car passes. Find the probability that she will complete the crossing before the next car passes. [2]

8 The numbers of a particular type of laptop computer sold by a store on each of 100 consecutive Saturdays are summarised in the following table.

Number sold	0	1	2	3	4	5	6	7	$\geqslant 8$
Number of Saturdays	7	20	39	16	14	2	1	1	0

Fit a Poisson distribution to the data and carry out a goodness of fit test at the 2.5% significance level. [9]

9 A random sample of 10 pairs of values of x and y is given in the following table.

x	4	6	6	8	2	7	12	14	9	5
y	2	4	6	8	6	10	9	8	6	5

(i) Find the equation of the regression line of y on x. [4]

(ii) Find the product moment correlation coefficient for the sample. [2]

(iii) Find the estimated value of y when $x = 10$, and comment on the reliability of this estimate. [2]

(iv) Another sample of N pairs of data from the same population has the same product moment correlation coefficient as the first sample given. A test, at the 1% significance level, on this second sample indicates that there is sufficient evidence to conclude that there is positive correlation. Find the set of possible values of N. [3]

10 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{2} & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^3$. Find the distribution function of Y. [5]

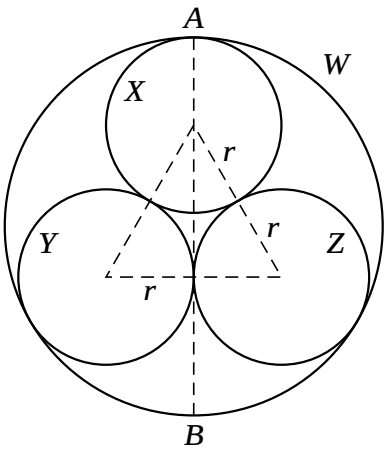
Sketch the graph of the probability density function of Y. [3]

Find the probability that Y lies between its median value and its mean value. [4]

© UCLES 2014

11 Answer only **one** of the following two alternatives.

EITHER



A uniform plane object consists of three identical circular rings, $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$, enclosed in a larger circular ring $\boldsymbol{W}$. Each of the inner rings has mass $\boldsymbol{m}$ and radius $\boldsymbol{r}$. The outer ring has mass $3\boldsymbol{m}$ and radius $\boldsymbol{R}$. The centres of the inner rings lie at the vertices of an equilateral triangle of side $2\boldsymbol{r}$. The outer ring touches each of the inner rings and the rings are rigidly joined together. The fixed axis $\boldsymbol{AB}$ is the diameter of $\boldsymbol{W}$ that passes through the centre of $\boldsymbol{X}$ and the point of contact of $\boldsymbol{Y}$ and $\boldsymbol{Z}$ (see diagram). It is given that $\boldsymbol{R} = \left(1 + \frac{2}{3}\sqrt{3}\right)\boldsymbol{r}$.

(i) Show that the moment of inertia of the object about $\boldsymbol{AB}$ is $\left(7 + 2\sqrt{3}\right)\boldsymbol{mr}^2$. [8]

The line $\boldsymbol{CD}$ is the diameter of $\boldsymbol{W}$ that is perpendicular to $\boldsymbol{AB}$. A particle of mass $9\boldsymbol{m}$ is attached to $\boldsymbol{D}$. The object is now held with its plane horizontal. It is released from rest and rotates freely about the fixed horizontal axis $\boldsymbol{AB}$.

(ii) Find, in terms of $\boldsymbol{g}$ and $\boldsymbol{r}$, the angular speed of the object when it has rotated through 60°. [6]

OR

Fish of a certain species live in two separate lakes, $\boldsymbol{A}$ and $\boldsymbol{B}$. A zoologist claims that the mean length of fish in $\boldsymbol{A}$ is greater than the mean length of fish in $\boldsymbol{B}$. To test his claim, he catches a random sample of 8 fish from $\boldsymbol{A}$ and a random sample of 6 fish from $\boldsymbol{B}$. The lengths of the 8 fish from $\boldsymbol{A}$, in appropriate units, are as follows.

15.3 12.0 15.1 11.2 14.4 13.8 12.4 11.8

Assuming a normal distribution, find a 95% confidence interval for the mean length of fish in $\boldsymbol{A}$. [5]

The lengths of the 6 fish from $\boldsymbol{B}$, in the same units, are as follows.

15.0 10.7 13.6 12.4 11.6 12.6

Stating any assumptions that you make, test at the 5% significance level whether the mean length of fish in $\boldsymbol{A}$ is greater than the mean length of fish in $\boldsymbol{B}$. [7]

Calculate a 95% confidence interval for the difference in the mean lengths of fish from $\boldsymbol{A}$ and from $\boldsymbol{B}$. [2]

© UCLES 2014

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.