UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

0088918998

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

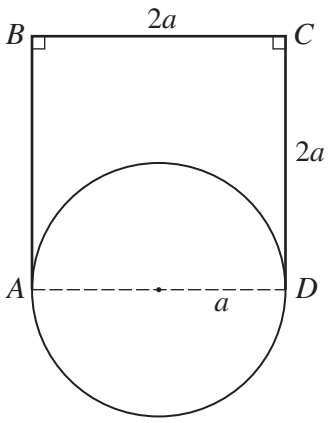
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

© UCLES 2013

1

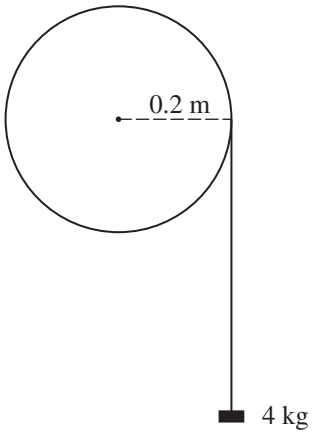


Three identical uniform rods, ***AB***, ***BC*** and ***CD***, each of mass ***M*** and length $2\boldsymbol{a}$, are rigidly joined to form three sides of a square. A uniform circular disc, of mass $\frac{2}{3}\boldsymbol{M}$ and radius $\boldsymbol{a}$, has the opposite ends of one of its diameters attached to ***A*** and ***D*** respectively. The disc and the rods all lie in the same plane (see diagram). Find the moment of inertia of the system about the axis ***AD***. [6]

2 The point ***O*** is on the fixed line ***l***. The point ***A*** on ***l*** is such that ***OA*** = 3 m. A particle ***P*** oscillates on ***l*** in simple harmonic motion with centre ***O*** and period π seconds. When ***P*** is at ***A*** its speed is $12\,\mathrm{m\,s^{-1}}$. Find the speed of ***P*** when it is at the point ***B*** on ***l***, where ***OB*** = 6 m and ***B*** is on the same side of ***O*** as ***A***. [4]

Find, correct to 2 decimal places, the time, in seconds, taken for ***P*** to travel directly from ***A*** to ***B***. [4]

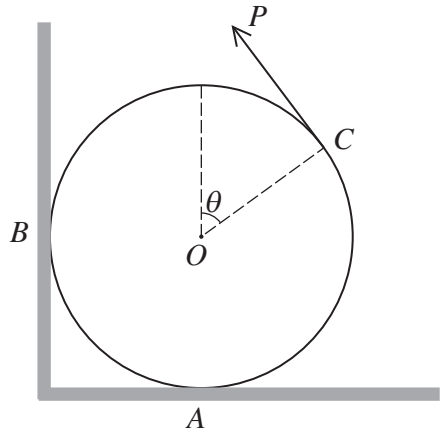
3



A uniform disc, of mass 2 kg and radius 0.2 m, is free to rotate in a vertical plane about a smooth horizontal axis through its centre. One end of a light inextensible string is attached to a point on the rim of the disc and the string is wound round the rim. The other end of the string is attached to a small block of mass 4 kg, which hangs freely (see diagram). The system is released from rest. During the subsequent motion, the block experiences a constant resistance to its motion, of magnitude ***R*** N. Given that the angular speed of the disc after it has turned through 2 radians is $5\,\mathrm{rad\,s^{-1}}$, find ***R*** and the tension in the string. [9]

© UCLES 2013

4



A uniform circular disc, with centre $\boldsymbol{O}$ and weight $\boldsymbol{W}$, rests in equilibrium on a horizontal floor and against a vertical wall. The plane of the disc is vertical and perpendicular to the wall. The disc is in contact with the floor at $\boldsymbol{A}$ and with the wall at $\boldsymbol{B}$. A force of magnitude $\boldsymbol{P}$ acts tangentially on the disc at the point $\boldsymbol{C}$ on the edge of the disc, where the radius $\boldsymbol{OC}$ makes an angle θ with the upward vertical, and $\tan\theta = \frac{4}{3}$ (see diagram). The coefficient of friction between the disc and the floor and between the disc and the wall is $\frac{1}{2}$. Show that the sum of the magnitudes of the frictional forces at $\boldsymbol{A}$ and $\boldsymbol{B}$ is equal to $\boldsymbol{P}$. [2]

Given that the equilibrium is limiting at both $\boldsymbol{A}$ and $\boldsymbol{B}$,

(i) show that $\boldsymbol{P} = \frac{15}{34}\boldsymbol{W}$, [6]

(ii) find the ratio of the magnitude of the normal reaction at $\boldsymbol{A}$ to the magnitude of the normal reaction at $\boldsymbol{B}$. [2]

5 Two uniform small smooth spheres $\boldsymbol{A}$ and $\boldsymbol{B}$, of equal radii, have masses $2\boldsymbol{m}$ and $\boldsymbol{m}$ respectively. They lie at rest on a smooth horizontal plane. Sphere $\boldsymbol{A}$ is projected directly towards $\boldsymbol{B}$ with speed $\boldsymbol{u}$. After the collision $\boldsymbol{B}$ goes on to collide directly with a fixed smooth vertical barrier, before colliding with $\boldsymbol{A}$ again. The coefficient of restitution between $\boldsymbol{A}$ and $\boldsymbol{B}$ is $\frac{2}{3}$ and the coefficient of restitution between $\boldsymbol{B}$ and the barrier is $\boldsymbol{e}$. After the second collision between $\boldsymbol{A}$ and $\boldsymbol{B}$, the speed of $\boldsymbol{B}$ is five times the speed of $\boldsymbol{A}$. Find the two possible values of $\boldsymbol{e}$. [11]

6 A fair die is thrown until a 5 or a 6 is obtained. The number of throws taken is denoted by the random variable $\boldsymbol{X}$. State the mean value of $\boldsymbol{X}$. [1]

Find the probability that obtaining a 5 or a 6 takes more than 8 throws. [2]

Find the least integer $\boldsymbol{n}$ such that the probability of obtaining a 5 or a 6 in fewer than $\boldsymbol{n}$ throws is more than 0.99. [3]

7 A random sample of 10 observations of a normally distributed random variable $\boldsymbol{X}$ gave the following summarised data, where $\bar{\boldsymbol{x}}$ denotes the sample mean.

$$\Sigma \boldsymbol{x} = 70.4 \qquad \Sigma (\boldsymbol{x} - \bar{\boldsymbol{x}})^2 = 8.48$$

Test, at the 10% significance level, whether the population mean of $\boldsymbol{X}$ is less than 7.5. [7]

© UCLES 2013

8 The lifetime, in years, of an electrical component is the random variable $\boldsymbol{T}$, with probability density function f given by

$$\mathrm{f}(\boldsymbol{t}) = \begin{cases} \boldsymbol{A}\mathrm{e}^{-\lambda \boldsymbol{t}} & \boldsymbol{t} \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $\boldsymbol{A}$ and λ are positive constants.

(i) Show that $\boldsymbol{A} = \lambda$. [2]

It is known that out of 100 randomly chosen components, 16 failed within the first year.

(ii) Find an estimate for the value of λ, and hence find an estimate for the median value of $\boldsymbol{T}$. [6]

9 For a random sample of 10 observations of pairs of values $(\boldsymbol{x}, \boldsymbol{y})$, the equations of the regression lines of $\boldsymbol{y}$ on $\boldsymbol{x}$ and of $\boldsymbol{x}$ on $\boldsymbol{y}$ are

$$\boldsymbol{y} = 4.21\boldsymbol{x} - 0.862 \qquad \text{and} \qquad \boldsymbol{x} = 0.043\boldsymbol{y} + 6.36,$$

respectively.

(i) Find the value of the product moment correlation coefficient for the sample. [3]

(ii) Test, at the 10% significance level, whether there is evidence of non-zero correlation between the variables. [4]

(iii) Find the mean values of $\boldsymbol{x}$ and $\boldsymbol{y}$ for this sample. [2]

(iv) Estimate the value of $\boldsymbol{x}$ when $\boldsymbol{y} = 2.3$ and comment on the reliability of your answer. [2]

10 Customers were asked which of three brands of coffee, $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$, they prefer. For a random sample of 80 male customers and 60 female customers, the numbers preferring each brand are shown in the following table.

	$\boldsymbol{A}$	$\boldsymbol{B}$	$\boldsymbol{C}$
Male	32	36	12
Female	18	30	12

Test, at the 5% significance level, whether there is a difference between coffee preferences of male and female customers. [8]

A larger random sample is now taken. It consists of $80\boldsymbol{n}$ male customers and $60\boldsymbol{n}$ female customers, where $\boldsymbol{n}$ is a positive integer. It is found that the proportions choosing each brand are identical to those in the smaller sample. Find the least value of $\boldsymbol{n}$ that would lead to a different conclusion for the 5% significance level hypothesis test. [4]

© UCLES 2013

11 Answer only **one** of the following two alternatives.

EITHER

A smooth sphere, with centre $\boldsymbol{O}$ and radius $\boldsymbol{a}$, is fixed on a smooth horizontal plane Π. A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ is projected horizontally from the highest point of the sphere with speed $\sqrt{\left(\frac{2}{5}\boldsymbol{ga}\right)}$. While $\boldsymbol{P}$ remains in contact with the sphere, the angle between $\boldsymbol{OP}$ and the upward vertical is denoted by θ. Show that $\boldsymbol{P}$ loses contact with the sphere when $\cos\theta = \frac{4}{5}$. [6]

Subsequently the particle collides with the plane Π. The coefficient of restitution between $\boldsymbol{P}$ and Π is $\frac{5}{9}$. Find the vertical height of $\boldsymbol{P}$ above Π when the vertical component of the velocity of $\boldsymbol{P}$ first becomes zero. [6]

OR

A factory produces bottles of spring water. The manager decides to assess the performance of the two machines that are used to fill the bottles with water. He selects a random sample of 60 bottles filled by the first machine $\boldsymbol{X}$ and a random sample of 80 bottles filled by the second machine $\boldsymbol{Y}$. The volumes of water, $\boldsymbol{x}$ and $\boldsymbol{y}$, measured in appropriate units, are summarised as follows.

$$\Sigma\boldsymbol{x} = 58.2 \qquad \Sigma\boldsymbol{x}^2 = 85.8 \qquad \Sigma\boldsymbol{y} = 97.6 \qquad \Sigma\boldsymbol{y}^2 = 188.6$$

A test at the $\alpha\%$ significance level shows that the mean volume of water in bottles filled by machine $\boldsymbol{X}$ is less than the mean volume of water in bottles filled by machine $\boldsymbol{Y}$. Find the set of possible values of α. [12]

© UCLES 2013

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.