UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **October/November 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

0251397462

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

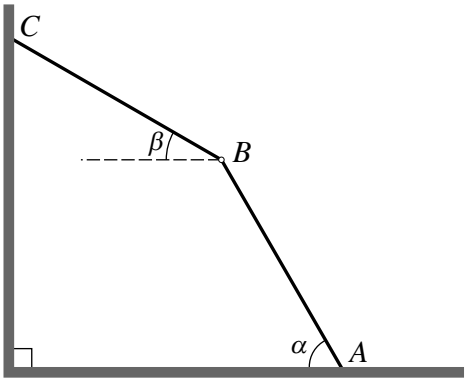
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1 A particle P is moving in a circle of radius 1.5 m. At time t s its velocity is $(k - t^2)\,\mathrm{m\,s^{-1}}$, where k is a positive constant. When $t = 3$, the magnitudes of the radial and transverse components of the acceleration of P are equal. Find the possible values of k. [4]

2 A small bead of mass m is threaded on a thin smooth wire which forms a circle of radius a. The wire is fixed in a vertical plane. A light inextensible string is attached to the bead and passes through a small smooth ring fixed at the centre of the circle. The other end of the string is attached to a particle of mass $4m$ which hangs freely under gravity. The bead is projected from the lowest point of the wire with speed $\sqrt{(kga)}$. Show that, when the angle between the two parts of the string is θ, the normal force exerted on the bead by the wire is $mg(3\cos\theta + k - 6)$, towards the centre. [5]

Given that the bead reaches the highest point of the wire, find an inequality which must be satisfied by k. [2]

3



Two uniform rods AB and BC, each of length $2a$ and mass m, are smoothly hinged at B. They rest in equilibrium with C in contact with a smooth vertical wall and A in contact with a rough horizontal floor. The rods are in a vertical plane perpendicular to the wall. The rods AB and BC make angles α and β respectively with the horizontal (see diagram). Show that

(i) the reaction at C has magnitude $\frac{1}{2}mg\cot\beta$, [2]

(ii) $\tan\alpha = 3\tan\beta$. [5]

The coefficient of friction at A is μ. Given that $\alpha = 60^\circ$, find the least possible value of μ. [2]

4 Three particles A, B and C have masses m, $2m$ and m respectively. The particles are able to move on a smooth horizontal surface in a straight line, and B is between A and C. Initially A is moving towards B with speed $2u$ and C is moving towards B with speed u. The particle B is at rest. The coefficient of restitution between any pair of particles is e. The first collision is between A and B.

(i) Show that the speed of B immediately before its collision with C is $\frac{2}{3}u(1 + e)$. [4]

(ii) Find the velocity of B immediately after its collision with C. [3]

(iii) Given that $e > \frac{1}{2}$, show that there are no further collisions between the particles. [4]

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5 Four identical uniform rods, each of mass m and length $2a$, are rigidly joined to form a square frame $ABCD$. Show that the moment of inertia of the frame about an axis through A perpendicular to the plane of the frame is $\frac{40}{3}ma^2$. [4]

The frame is suspended from A and is able to rotate freely under gravity in a vertical plane, about a horizontal axis through A. When the frame is at rest with C vertically below A, it is given an angular velocity $\sqrt{\left(\frac{6g}{5a}\right)}$. Find the angular velocity of the frame when AC makes an angle θ with the downward vertical through A. [5]

When AC is horizontal, the speed of C is $k\sqrt{(ga)}$. Find the value of k correct to 3 significant figures. [3]

6 The random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}\mathrm{e}^{-\frac{1}{6}x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Find

(i) the distribution function of X, [2]

(ii) the probability that X lies between the median and the mean. [4]

7 The speed v at which a javelin is thrown by an athlete is measured in $\mathrm{km\,h^{-1}}$. The results for 10 randomly chosen throws are summarised by

$$\Sigma v = 1110.8, \quad \Sigma(v - \bar{v})^2 = 333.9,$$

where $\bar{v}$ is the sample mean.

(i) Stating any necessary assumption, calculate a 99% confidence interval for the mean speed of a throw. [6]

The results for a further 5 randomly chosen throws are now combined with the above results. It is found that the sample variance is smaller than that used in part **(i)**.

(ii) State, with reasons, whether a 95% confidence interval calculated from the combined 15 results will be wider or less wide than that found in part **(i)**. [2]

8 Drinking glasses are sold in packs of 4. The manufacturer conducts a survey to assess the quality of the glasses. The results from a sample of 50 randomly chosen packs are summarised in the following table.

Number of perfect glasses	0	1	2	3	4
Number of packs	1	3	10	17	19

Fit a binomial distribution to the data and carry out a goodness of fit test at the 10% significance level. [9]

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9 Experiments are conducted to test the breaking strength of each of two types of rope, P and Q. A random sample of 50 ropes of type P and a random sample of 70 ropes of type Q are selected. The breaking strengths, p and q, measured in appropriate units, are summarised as follows.

$$\Sigma p = 321.2 \qquad \Sigma p^2 = 2120.0 \qquad \Sigma q = 475.3 \qquad \Sigma q^2 = 3310.0$$

Test, at the 10% significance level, whether the mean breaking strengths of type P and type Q ropes are the same. [10]

10 Delegates who travelled to a conference were asked to report the distance, y km, that they had travelled and the time taken, x minutes. The values reported by a random sample of 8 delegates are given in the following table.

Delegate	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
x	90	46	72	98	52	65	105	82
y	90	55	69	85	45	50	110	74

$[\Sigma x = 610,\ \Sigma x^2 = 49\,682,\ \Sigma y = 578,\ \Sigma y^2 = 45\,212,\ \Sigma xy = 47\,136.]$

Find the equations of the regression lines of y on x and of x on y. [6]

Estimate the time taken by a delegate who travelled 100 km to the conference. [2]

Calculate the product moment correlation coefficient for this sample. [2]

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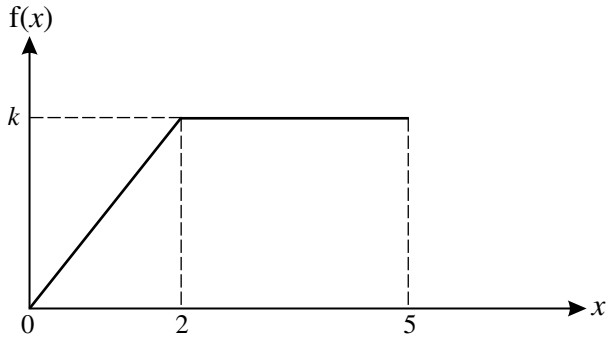
11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light elastic string of modulus of elasticity $8mg$ and natural length a. The other end of the string is attached to a fixed point O. The particle is pulled vertically downwards a distance $\frac{1}{4}a$ from its equilibrium position and released from rest. Show that the string first becomes slack after a time $\frac{2\pi}{3}\sqrt{\left(\frac{a}{8g}\right)}$. [8]

Find, in terms of a, the total distance travelled by P from its release until it subsequently comes to instantaneous rest for the first time. [6]

OR



The continuous random variable X takes values in the interval $0 \leqslant x \leqslant 5$ only. For $0 \leqslant x \leqslant 5$ the graph of its probability density function f consists of two straight line segments, as shown in the diagram. Find k and show that f is given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{8}x & 0 \leqslant x \leqslant 2, \\ \frac{1}{4} & 2 < x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$ [3]

The random variable Y is given by $Y = X^2$.

(i) Find the probability density function of Y. [6]

(ii) Show that $\mathrm{E}(Y) = 10.25$. [3]

(iii) Show that the median of Y is the square of the median of X. [2]

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