UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 5 8 4 8 7 7 9 6 9 7 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

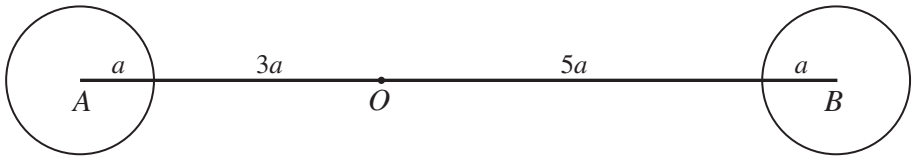
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

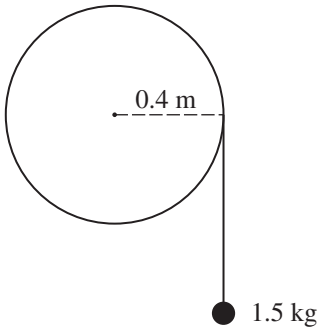
© UCLES 2012

1



A rigid body consists of two uniform circular discs, each of mass $\boldsymbol{m}$ and radius $\boldsymbol{a}$, the centres of which are rigidly attached to the ends $\boldsymbol{A}$ and $\boldsymbol{B}$ of a uniform rod of mass $3\boldsymbol{m}$ and length $10\boldsymbol{a}$. The discs and the rod are in the same plane and $\boldsymbol{O}$ is the point on the rod such that $\boldsymbol{AO} = 4\boldsymbol{a}$ (see diagram). Show that the moment of inertia of the body about an axis through $\boldsymbol{O}$ perpendicular to the plane of the discs is $81\boldsymbol{ma}^2$. [5]

2



A uniform disc of radius 0.4 m is free to rotate without friction in a vertical plane about a horizontal axis through its centre. The moment of inertia of the disc about the axis is $0.2\,\text{kg}\,\text{m}^2$. One end of a light inextensible string is attached to a point on the rim of the disc and the string is wound round the rim. The other end of the string is attached to a particle of mass 1.5 kg which hangs freely (see diagram). The system is released from rest. Find

(i) the angular acceleration of the disc, [4]

(ii) the speed of the particle when the disc has turned through an angle of $\frac{1}{6}\pi$. [3]

3 A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ is attached to one end of a light inextensible string of length $\boldsymbol{a}$. The other end of the string is attached to a fixed point $\boldsymbol{O}$. The particle is held with the string taut and horizontal and is then released. When the string is vertical, it comes into contact with a small smooth peg $\boldsymbol{A}$ which is vertically below $\boldsymbol{O}$ and at a distance $\boldsymbol{x}$ ($< \boldsymbol{a}$) from $\boldsymbol{O}$. In the subsequent motion, when $\boldsymbol{AP}$ makes an angle θ with the downward vertical, the tension in the string is $\boldsymbol{T}$. Show that

$$\boldsymbol{T} = \boldsymbol{mg}\left(3\cos\theta + \frac{2\boldsymbol{x}}{\boldsymbol{a} - \boldsymbol{x}}\right).$$ [7]

Given that $\boldsymbol{P}$ completes a vertical circle about $\boldsymbol{A}$, find the least possible value of $\dfrac{\boldsymbol{x}}{\boldsymbol{a}}$. [2]

© UCLES 2012

4 A particle $\boldsymbol{P}$ of mass $2\boldsymbol{m}$, moving on a smooth horizontal plane with speed $\boldsymbol{u}$, strikes a fixed smooth vertical barrier. Immediately before the collision the angle between the direction of motion of $\boldsymbol{P}$ and the barrier is 60°. The coefficient of restitution between $\boldsymbol{P}$ and the barrier is $\frac{1}{3}$. Show that $\boldsymbol{P}$ loses two-thirds of its kinetic energy in the collision. [5]

Subsequently $\boldsymbol{P}$ collides directly with a particle $\boldsymbol{Q}$ of mass $\boldsymbol{m}$ which is moving on the plane with speed $\boldsymbol{u}$ towards $\boldsymbol{P}$. The magnitude of the impulse acting on each particle in the collision is $\frac{2}{3}\boldsymbol{mu}(1+\sqrt{3})$.

(i) Show that the speed of $\boldsymbol{P}$ after this collision is $\frac{1}{3}\boldsymbol{u}$. [2]

(ii) Find the exact value of the coefficient of restitution between $\boldsymbol{P}$ and $\boldsymbol{Q}$. [4]

5 A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ lies on a smooth horizontal surface. $\boldsymbol{A}$ and $\boldsymbol{B}$ are fixed points on the surface, where $\boldsymbol{AB} = 10\boldsymbol{a}$. A light elastic string, of natural length $2\boldsymbol{a}$ and modulus of elasticity $8\boldsymbol{mg}$, joins $\boldsymbol{P}$ to $\boldsymbol{A}$. Another light elastic string, of natural length $4\boldsymbol{a}$ and modulus of elasticity $16\boldsymbol{mg}$, joins $\boldsymbol{P}$ to $\boldsymbol{B}$. Show that when $\boldsymbol{P}$ is in equilibrium, $\boldsymbol{AP} = 4\boldsymbol{a}$. [3]

The particle is held at rest at the point $\boldsymbol{C}$ between $\boldsymbol{A}$ and $\boldsymbol{B}$ on the line $\boldsymbol{AB}$ where $\boldsymbol{AC} = 3\boldsymbol{a}$. The particle is now released.

(i) Show that the subsequent motion of $\boldsymbol{P}$ is simple harmonic with period $\pi\sqrt{\left(\frac{\boldsymbol{a}}{2\boldsymbol{g}}\right)}$. [6]

(ii) Find the maximum speed of $\boldsymbol{P}$. [2]

6 In a skiing resort, for each day during the winter season, the probability that snow will fall on that day is 0.2, independently of any other day. The first day of the winter season is 1 December. Find, for the winter season,

(i) the probability that the first snow falls on 20 December, [2]

(ii) the probability that the first snow falls before 5 December, [2]

(iii) the earliest date in December such that the probability that the first snow falls on or before that date is at least 0.95. [3]

7 The continuous random variable $\boldsymbol{X}$ has probability density function f given by

$$\mathrm{f}(\boldsymbol{x}) = \begin{cases} \frac{2}{15}\boldsymbol{x} & 1 \leqslant \boldsymbol{x} \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable $\boldsymbol{Y}$ is defined by $\boldsymbol{Y} = \boldsymbol{X}^3$. Show that the distribution function G of $\boldsymbol{Y}$ is given by

$$\mathrm{G}(\boldsymbol{y}) = \begin{cases} 0 & \boldsymbol{y} < 1, \\ \frac{1}{15}\left(\boldsymbol{y}^{\frac{2}{3}} - 1\right) & 1 \leqslant \boldsymbol{y} \leqslant 64, \\ 1 & \boldsymbol{y} > 64. \end{cases}$$ [4]

Find

(i) the median value of $\boldsymbol{Y}$, [3]

(ii) $\mathrm{E}(\boldsymbol{Y})$. [4]

© UCLES 2012

8 The yield of a particular crop on a farm is thought to depend principally on the amount of sunshine during the growing season. For a random sample of 8 years, the average yield, y kilograms per square metre, and the average amount of sunshine per day, x hours, are recorded. The results are given in the following table.

x	12.2	10.4	5.2	6.3	11.8	10.0	14.2	2.3
y	15	9	10	7	8	11	12	6

[$\Sigma x = 72.4$, $\Sigma x^2 = 769.9$, $\Sigma y = 78$, $\Sigma y^2 = 820$, $\Sigma xy = 761.3$.]

(i) Find the equation of the regression line of y on x. [4]

(ii) Find the product moment correlation coefficient. [3]

(iii) Test, at the 5% significance level, whether there is positive correlation between the average yield and the average amount of sunshine per day. [4]

9 The leaves from oak trees growing in two different areas A and B are being measured. The lengths, in cm, of a random sample of 7 oak leaves from area A are

6.2, 8.3, 7.8, 9.3, 10.2, 8.4, 7.2.

Assuming that the distribution is normal, find a 95% confidence interval for the mean length of oak leaves from area A. [5]

The lengths, in cm, of a random sample of 5 oak leaves from area B are

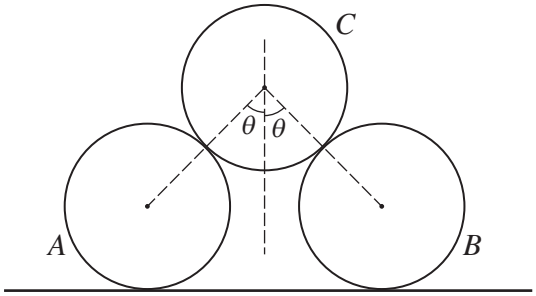
5.9, 7.4, 6.8, 8.2, 8.7.

Making suitable assumptions, which should be stated, test, at the 5% significance level, whether the mean length of oak leaves from area A is greater than the mean length of oak leaves from area B. [9]

© UCLES 2012

10 Answer only **one** of the following two alternatives.

EITHER



Two identical uniform rough spheres $\boldsymbol{A}$ and $\boldsymbol{B}$, each of weight $\boldsymbol{W}$ and radius $\boldsymbol{a}$, are at rest on a rough horizontal plane, and are not in contact with each other. A third identical sphere $\boldsymbol{C}$ rests on $\boldsymbol{A}$ and $\boldsymbol{B}$ with its centre in the same vertical plane as the centres of $\boldsymbol{A}$ and $\boldsymbol{B}$. The line joining the centres of $\boldsymbol{A}$ and $\boldsymbol{C}$ and the line joining the centres of $\boldsymbol{B}$ and $\boldsymbol{C}$ are each inclined at an angle θ to the vertical (see diagram). The coefficient of friction between each sphere and the plane is μ. The coefficient of friction between $\boldsymbol{C}$ and $\boldsymbol{A}$, and between $\boldsymbol{C}$ and $\boldsymbol{B}$, is μ'. The system remains in equilibrium. Show that

$$\mu \geqslant \frac{\sin\theta}{3(1+\cos\theta)} \quad \text{and} \quad \mu' \geqslant \frac{\sin\theta}{1+\cos\theta}. \qquad [14]$$

OR

A continuous random variable $\boldsymbol{X}$ is believed to have the probability density function f given by

$$\mathrm{f}(\boldsymbol{x}) = \begin{cases} \frac{3}{10}(5\boldsymbol{x} - \boldsymbol{x}^2 - 4) & 2 \leqslant \boldsymbol{x} < 4, \\ 0 & \text{otherwise.} \end{cases}$$

A random sample of 60 observations was taken and these values are summarised in the following grouped frequency table.

Interval	$2 \leqslant \boldsymbol{x} < 2.4$	$2.4 \leqslant \boldsymbol{x} < 2.8$	$2.8 \leqslant \boldsymbol{x} < 3.2$	$3.2 \leqslant \boldsymbol{x} < 3.6$	$3.6 \leqslant \boldsymbol{x} < 4$
Observed frequency	19	17	16	8	0

The estimated mean, based on the grouped data in the table above, is 2.69, correct to 2 decimal places. It is decided that a goodness of fit test will only be conducted if the mean predicted from the probability density function is within 10% of the estimated mean. Show that this condition is satisfied. [5]

The relevant expected frequencies are as follows.

Interval	$2 \leqslant \boldsymbol{x} < 2.4$	$2.4 \leqslant \boldsymbol{x} < 2.8$	$2.8 \leqslant \boldsymbol{x} < 3.2$	$3.2 \leqslant \boldsymbol{x} < 3.6$	$3.6 \leqslant \boldsymbol{x} < 4$
Expected frequency	15.456	16.032	14.304	10.272	3.936

Show how the expected frequency for the interval $3.2 \leqslant \boldsymbol{x} < 3.6$ is obtained. [2]

Carry out the goodness of fit test at the 10% significance level. [7]

© UCLES 2012

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.