UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2 **October/November 2010**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 7 8 4 2 4 7 8 6 8 6 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

© UCLES 2010

1 A particle P is describing simple harmonic motion of amplitude 5 m. Its speed is $6\,\text{m}\,\text{s}^{-1}$ when it is 3 m from the centre of the motion. Find, in terms of π, the period of the motion. [2]

Find also

(i) the maximum speed of P, [2]

(ii) the magnitude of the maximum acceleration of P. [2]

2 A particle P of mass m is projected horizontally with speed u from the lowest point on the inside of a fixed hollow sphere with centre O. The sphere has a smooth internal surface of radius a. Assuming that the particle does not lose contact with the sphere, show that when the speed of the particle has been reduced to $\frac{1}{2}u$ the angle θ between OP and the downward vertical satisfies the equation

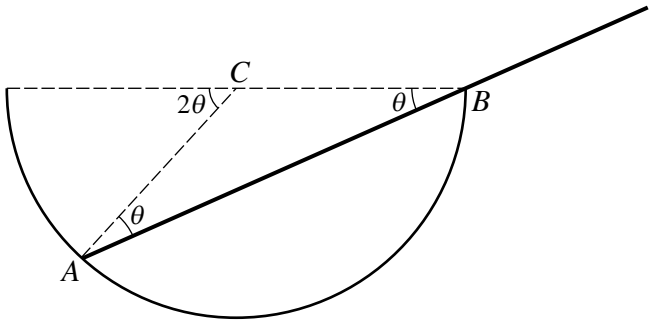
$$8ga(1 - \cos\theta) = 3u^2. \quad [2]$$

Find, in terms of m, u, a and g, an expression for the magnitude of the contact force acting on the particle in this position. [4]

3 Two smooth spheres A and B, of equal radius, are moving in the same direction in the same straight line on a smooth horizontal table. Sphere A has mass m and speed u and sphere B has mass αm and speed $\frac{1}{4}u$. The spheres collide and A is brought to rest by the collision. Find the coefficient of restitution in terms of α. [6]

Deduce that $\alpha \geqslant 2$. [2]

4



A hemispherical bowl of radius r is fixed with its rim horizontal. A thin uniform rod rests in equilibrium on the rim of the bowl with one end resting on the inner surface of the bowl at A, as shown in the diagram. The rod has length $2a$ and weight W. The point of contact between the rod and the rim is B, and the rim has centre C. The rod is in a vertical plane containing C. The rod is inclined at θ to the horizontal and the line AC is inclined at 2θ to the horizontal. The contacts at A and B are smooth.

In any order, show that

(i) the contact force acting on the rod at A has magnitude $W\tan\theta$,

(ii) the contact force acting on the rod at B has magnitude $\dfrac{W\cos 2\theta}{\cos\theta}$,

(iii) $2r\cos 2\theta = a\cos\theta$.

[9]

© UCLES 2010

5 A uniform circular disc has diameter AB, mass $2m$ and radius a. A particle of mass m is attached to the disc at B. The disc is able to rotate about a smooth fixed horizontal axis through A. The axis is tangential to the disc. Show that the moment of inertia of the system about the axis is $\frac{13}{2}ma^2$. [4]

The disc is held with AB horizontal and released. Find the angular speed of the system when B is directly below A. [5]

The disc is slightly displaced from the position of equilibrium in which B is below A. At time t the angle between AB and the vertical is θ. Write down the equation of motion, and find the approximate period of small oscillations about the equilibrium position. [5]

6 The mean Intelligence Quotient (IQ) of a random sample of 15 pupils at School A is 109. The mean IQ of a random sample of 20 pupils at School B is 112. You may assume that the IQs for the populations from which these samples are taken are normally distributed, and that both distributions have standard deviation 15. Find a 90% confidence interval for $\mu_B - \mu_A$, where μ_A and μ_B are the population mean IQs. [6]

7 The discrete random variable X has a geometric distribution with mean 4.

Find

(i) $P(X = 5)$, [3]

(ii) $P(X \geqslant 5)$, [2]

(iii) the least integer N such that $P(X \leqslant N) > 0.9995$. [2]

8 The owner of three driving schools, A, B and C, wished to assess whether there was an association between passing the driving test and the school attended. He selected a random sample of learner drivers from each of his schools and recorded the numbers of passes and failures at each school. The results that he obtained are shown in the table below.

	Driving school attended		
	A	*B*	*C*
Passes	23	15	17
Failures	27	25	43

Using a χ^2-test and a 5% level of significance, test whether there is an association between passing or failing the driving test and the driving school attended. [7]

© UCLES 2010

9 A national athletics coach suspects that, on average, 200-metre runners' indoor times exceed their outdoor times by more than 0.1 seconds. In order to test this, the coach randomly selects eight 200-metre runners and records their indoor and outdoor times. The results, in seconds, are shown in the table.

Runner	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Indoor time	21.5	21.8	20.9	21.2	21.4	21.4	21.2	21.0
Outdoor time	21.1	21.7	20.7	20.9	21.3	21.0	21.1	20.8

Stating suitable hypotheses and any necessary assumption that you make, test the coach's suspicion at the 2.5% level of significance. [10]

10 For each month of a certain year, a weather station recorded the average rainfall per day, x mm, and the average amount of sunshine per day, y hours. The results are summarised below.

$$n = 12, \quad \Sigma x = 24.29, \quad \Sigma x^2 = 50.146, \quad \Sigma y = 45.8, \quad \Sigma y^2 = 211.16, \quad \Sigma xy = 88.415.$$

(i) Find the mean values, $\bar{x}$ and $\bar{y}$. [1]

(ii) Calculate the gradient of the line of regression of y on x. [2]

(iii) Use the answers to parts **(i)** and **(ii)** to obtain the equation of the line of regression of y on x. [2]

(iv) Find the product moment correlation coefficient and comment, in context, on its value. [4]

(v) Stating your hypotheses, test at the 1% level of significance whether there is negative correlation between average rainfall per day and average amount of sunshine per day. [4]

© UCLES 2010

11 Answer only **one** of the following two alternatives.

EITHER

A particle of mass 0.1 kg lies on a smooth horizontal table on the line between two points A and B on the table, which are 6 m apart. The particle is joined to A by a light elastic string of natural length 2 m and modulus of elasticity 60 N, and to B by a light elastic string of natural length 1 m and modulus of elasticity 20 N. The mid-point of AB is M, and O is the point between M and B at which the particle can rest in equilibrium. Show that $MO = 0.2$ m. [4]

The particle is held at M and then released. Show that the equation of motion is

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = -500y,$$

where y metres is the displacement from O in the direction OB at time t seconds, and state the period of the motion. [5]

For the instant when the particle is 0.3 m from M for the first time, find

(i) the speed of the particle, [2]

(ii) the time taken, after release, to reach this position. [3]

OR

The continuous random variable T has a negative exponential distribution with probability density function given by

$$\mathrm{f}(t) = \begin{cases} \lambda \mathrm{e}^{-\lambda t} & t \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Show that for $t \geqslant 0$ the distribution function is given by $\mathrm{F}(t) = 1 - \mathrm{e}^{-\lambda t}$. [2]

The table below shows some values of $\mathrm{F}(t)$ for the case when the mean is 20. Find the missing value. [2]

t	0	5	10	15	20	25	30	35	40
$\mathrm{F}(t)$	0	0.2212	0.3935		0.6321	0.7135	0.7769	0.8262	0.8647

It is thought that the lifetime of a species of insect under laboratory conditions has a negative exponential distribution with mean 20 hours. When observation starts there are 100 insects, which have been randomly selected. The lifetimes of the insects, in hours, are summarised in the table below.

Lifetime (hours)	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	⩾40
Frequency	20	20	11	9	9	8	5	1	17

Calculate the expected values for each interval, assuming a negative exponential model with a mean of 20 hours, giving your values correct to 2 decimal places. [3]

Perform a χ^2-test of goodness of fit, at the 5% level of significance, in order to test whether a negative exponential distribution, with a mean of 20 hours, is a suitable model for the lifetime of this species of insect under laboratory conditions. [7]

© UCLES 2010

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.