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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2025**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of θ to the downward vertical.

Given that $\tan\theta = \frac{4}{3}$, find ω in terms of a and g. [3]

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity mg. The other end of the string is attached to a fixed point O on a rough plane inclined at an angle of 30° to the horizontal. The particle P is held at rest at point O before being released. The frictional force acting on P as it slides down the plane is $\frac{11}{30}mg$.

(a) Find, in terms of a, the distance that P moves down the plane before coming to rest. [5]

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(b) It is given that P remains at rest in this new position.

Find, in terms of m and g, the magnitude of the frictional force in this position. [3]

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3 A ball of mass m kg is projected vertically upwards with initial speed U m s^{-1} and moves under gravity. At time t s after projection, the ball has travelled a distance x m and its speed is v m s^{-1}. There is a resistive force of magnitude mkv^2 N, where k is a positive constant.

(a) Show that the distance travelled by the ball when it is moving upwards is $x = \frac{1}{2k}\ln\left(\frac{g+kU^2}{g+kv^2}\right)$. [4]

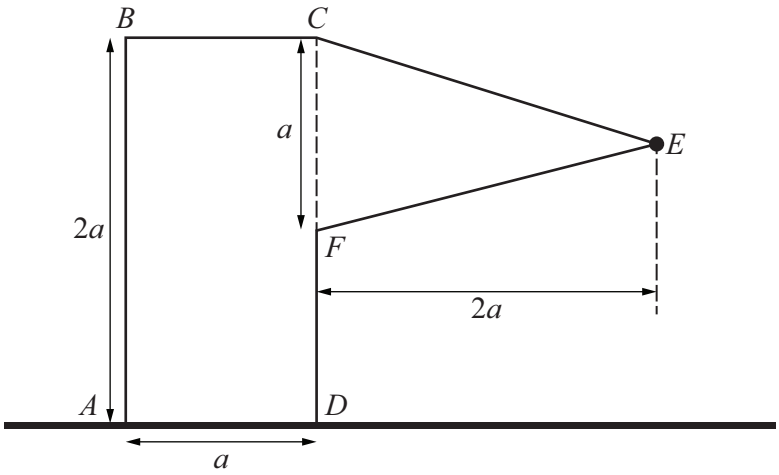
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It is given that $k = 0.025$ and that $U = 20$.

(b) Find the time taken for the ball to reach its maximum height. [4]

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An object consists of a uniform lamina with a particle attached. The uniform lamina $ABCEFD$ of mass m is formed from a rectangle $ABCD$ and an isosceles triangle CEF, where F is the midpoint of CD. The rectangle has sides $AB = 2a$ and $AD = a$. The triangle CEF has base a and height $2a$. The particle of mass km is attached to the lamina at E. The object rests in a vertical plane with its edge AD on horizontal ground (see diagram).

Given that the object is on the point of toppling in its vertical plane about the vertex D, find the value of k. [4]

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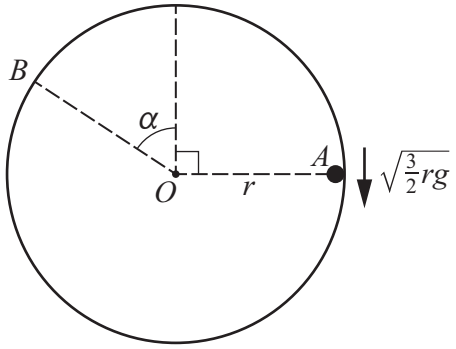
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A hollow cylinder of radius r is fixed with its axis horizontal. Points A, B and O are in the same vertical plane perpendicular to the axis of the cylinder, with A and B on the smooth inner surface and O on the axis. OA and OB make angles 90° and α respectively with the upward vertical through O, with A and B on opposite sides of the vertical. A particle of mass m is projected vertically downwards from point A with speed $\sqrt{\frac{3}{2}rg}$ and moves in a vertical circle inside the cylinder (see diagram). The particle loses contact with the cylinder at point B.

(a) Find the value of α. [4]

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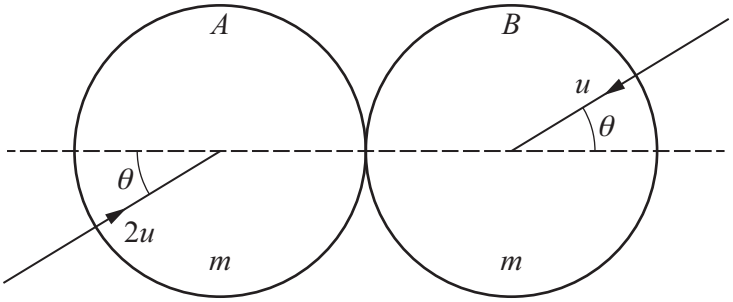
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(b) In the subsequent motion find, in terms of r, the greatest height above O reached by the particle. [4]

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Two identical uniform smooth spheres A and B, each with mass m, are moving on a horizontal surface with speeds $2u$ and u respectively when they collide. Immediately before the collision, the spheres are moving parallel to each other in opposite directions such that their directions of motion each make an angle θ with the line of centres (see diagram). As a result of the collision, B moves in a direction which is perpendicular to its initial direction of motion. The coefficient of restitution between the spheres is e.

(a) Find an expression for $\tan\theta$ in terms of e. [6]

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As a result of the collision, A moves in a direction which is perpendicular to the line of centres.

(b) Find the value of θ. [2]

7 A particle P is projected from a point O with speed U at an angle 45° above the horizontal and moves freely under gravity.

(a) State the vertical and horizontal components of velocity at time t. [1]

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At time T, particle P is moving at an angle of 60° below the horizontal.

(b) Show that $T = \frac{U}{2g}\left(\sqrt{2} + \sqrt{6}\right)$. [3]

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At time T, the particle strikes a smooth horizontal plane at a point which is a horizontal distance D from O and a vertical distance H below O.

(c) Find the ratio $H : D$. [4]

After striking the horizontal plane, P rebounds with speed w. The coefficient of restitution between P and the plane is $\frac{2}{3}$.

(d) Find w in terms of U. [3]

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Additional page

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