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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4412827330

FURTHER MATHEMATICS **9231/24**

Paper 2 Further Pure Mathematics 2 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Find the values of k for which the system of equations

$$\begin{aligned} x + 2y + 3z &= 1, \\ kx + 5y + 6z &= 2, \\ 7x + 2ky + 9z &= 3, \end{aligned}$$

does not have a unique solution. [3]

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(b) Given that $k = 1$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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2 Find the exact value of $\int_1^{\frac{5}{2}} \frac{1}{\sqrt{x^2-2x+5}}\,\mathrm{d}x$, giving your answer in logarithmic form. [6]

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3 Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4\frac{\mathrm{d}y}{\mathrm{d}x}+5y=13\mathrm{e}^{3x}$$

given that $y=1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=0$. [10]

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4 A curve has parametric equations

$$x = t^3 - t^2 + t - 1 \qquad \text{and} \qquad y = te^t.$$

(a) Show that 1 is the only real value of t for which $x = 0$. [1]

(b) Show that $\frac{dy}{dx} = \frac{(t+1)e^t}{3t^2 - 2t + 1}$. [3]

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(c) Find the Maclaurin's series for y up to and including the term in x^2. [6]

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5 **(a)** Use de Moivre's theorem to show that

$$\sin 7\theta = -64\sin^7\theta + 112\sin^5\theta - 56\sin^3\theta + 7\sin\theta.$$ [5]

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(b) Hence find all roots of the equation

$$64x^6 - 112x^4 + 56x^2 - 7 = 0$$

in the form $\sin q\pi$, where q is a rational number. [3]

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6 Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = 2x^2 \tan^{-1} x$$

for which $y = \frac{1}{2}\pi$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. [9]

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7 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 7 & 11 \\ 0 & 2 & 5 \\ 0 & 0 & -3 \end{pmatrix}.$$

(a) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^6 = \mathbf{PDP}^{-1}$. [7]

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(b) Use the characteristic equation of **A** to show that

$$\mathbf{A}^6 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I},$$

where a, b and c are integers to be determined. [4]

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8 The curve C has equation $y = \tanh x$ for $x \geqslant 0$.

(a) Sketch C and state the equation of the asymptote. [2]

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(b) By considering a suitable set of N rectangles of unit width, use your sketch to show that

$$\sum_{r=1}^{N} \tanh r > \ln(\cosh N).$$ [3]

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(c) The arc of C joining the point where $x = 0$ to the point where $x = \frac{1}{2}\ln 3$ is rotated through one complete revolution about the x-axis. The area of the surface generated is denoted by S.

(i) Use the substitution $u = \sqrt{1 + \text{sech}^4 x}$ to show that

$$S = \pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2 - 1} \, \mathrm{d}u. \qquad [7]$$

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(ii) Find the exact value of $\pi \int_{\frac{5}{4}}^{\sqrt{2}} \frac{u^2}{u^2-1} \mathrm{d}u$. You need not simplify your answer. [3]

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