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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3120670749

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **May/June 2025**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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[Turn over

1 Find the Maclaurin's series for $e^{\left(\frac{1}{x+2}\right)}$ up to and including the term in x^2. [5]

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2 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$\tanh^2 t + \operatorname{sech}^2 t = 1. \qquad [3]$$

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(b) The curve C has parametric equations

$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leqslant t \leqslant 1.$$

Find the length of C. [5]

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3 The curve C has equation

$$9y^2 - 3\sinh^{-1}(xy) = 1 - 3\ln 3.$$

(a) Show that, at the point $(4, \frac{1}{3})$ on C, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2}$. [4]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(4, \frac{1}{3})$. [5]

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4 Find the particular solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} - 2x = 2t^2 + t - 1,$$

given that, when $t = 0$, $x = \frac{\mathrm{d}x}{\mathrm{d}t} = 0$. [10]

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5 **(a)** Use de Moivre's theorem to show that

$$\sec 5\theta = \frac{\sec^5\theta}{5\sec^4\theta - 20\sec^2\theta + 16}.$$ [6]

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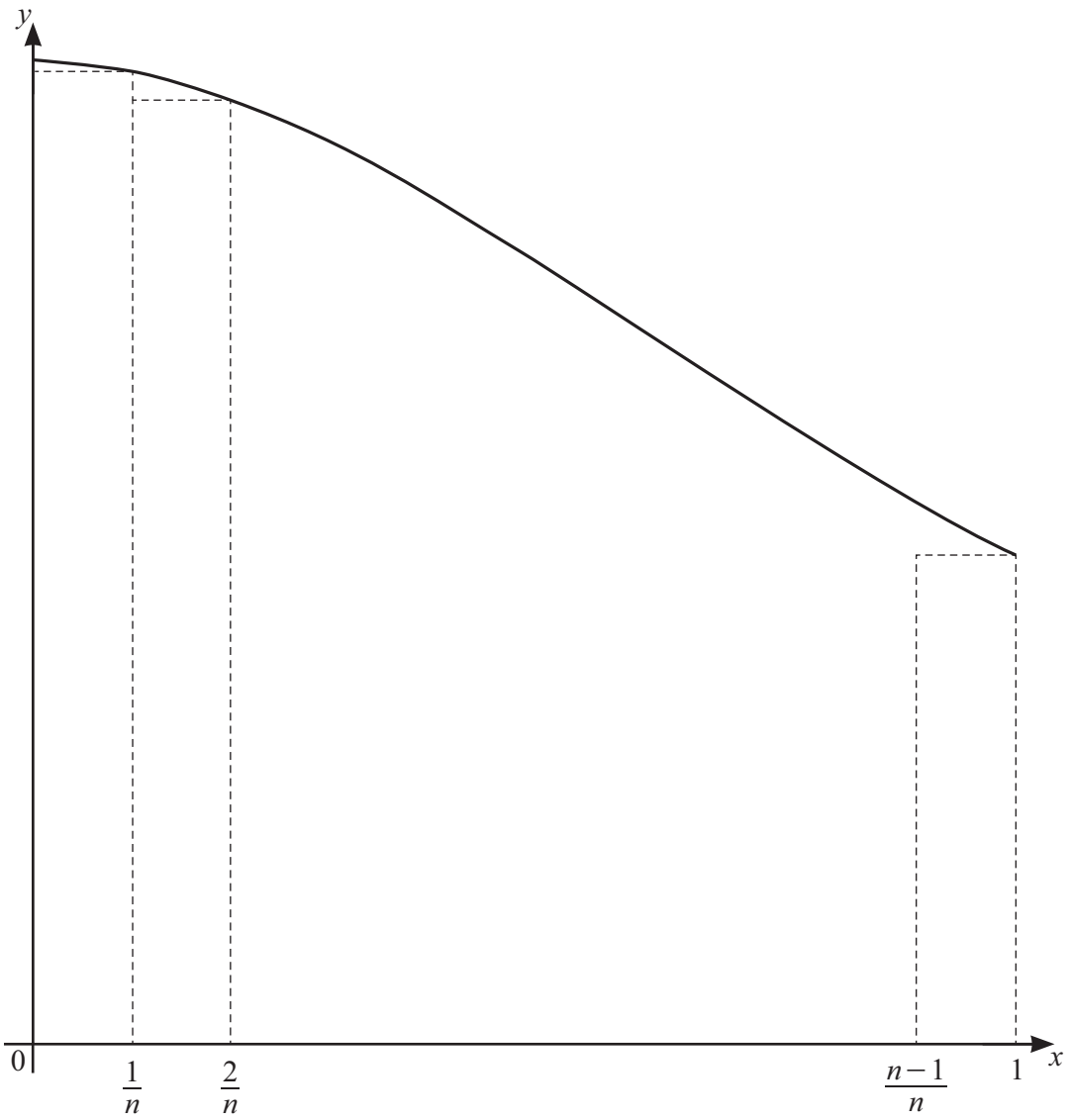
(b) Hence, obtain the roots of the equation

$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\sec(q\pi)$, where q is rational. [4]

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The diagram shows the curve with equation $y = \dfrac{1}{x^2+1}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\dfrac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{n}{n^2+r^2} < \frac{1}{4}\pi. \qquad [5]$$

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(b) Use a similar method to find a lower bound for $\sum_{r=1}^{n} \frac{n}{n^2+r^2}$. Give your answer in terms of n and π. [4]

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(c) Deduce the exact value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$. [1]

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7 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2x+6}{x^2+6x+5}y = 4,$$

given that $y = 0$ when $x = 0$. Give your answer in an exact form. [9]

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8 **(a)** Find the values of a for which the system of equations

$$\begin{aligned} \tfrac{3}{2}x + 3y + 8z &= 1, \\ ax + 3y + 4z &= 2, \\ ay - z &= 3, \end{aligned}$$

does not have a unique solution. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

(b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

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(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

Additional page

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