* 0019655481101 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8661254792

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

© UCLES 2024

[Turn over

1 A college uses two assessments, X and Y, when interviewing applicants for research posts at the college. These assessments have been used for a large number of applicants this year.

The scores for a random sample of 9 applicants who took assessment X are as follows.

21.4 24.6 25.3 22.7 20.8 21.5 22.9 21.3 22.3

The scores for a random sample of 10 applicants who took assessment Y are as follows.

20.9 23.5 24.8 21.9 23.4 24.0 23.8 24.1 25.1 25.8

The interviewer believes that the population median score from assessment X is lower than the population median score from assessment Y.

Carry out a Wilcoxon rank-sum test, at the 1% significance level, to test whether the interviewer's belief is supported by the data. [7]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

2 A rowing club has a large number of members. A random sample of 12 of these members is taken and the pulse rate, x beats per minute (bpm), of each is measured after a 30-minute training session. A 98% confidence interval for the population mean pulse rate, μ bpm, is calculated from the sample as $64.22 < \mu < 68.66$.

(a) Find the values of Σx and Σx^2. [6]

(b) State an assumption that is necessary for the confidence interval to be valid. [1]

DO NOT WRITE IN THIS MARGIN

3 There are three bus companies in a city. The council is investigating whether the buses reliably arrive at their destination on time. The results from random samples of buses from each company are summarised in the following table.

		Bus company			
		A	*B*	*C*	Total
Arrival	Early	22	22	10	54
	On time	30	52	42	124
	Late	28	26	18	72
	Total	80	100	70	250

Test, at the 5% significance level, whether the reliability of buses is independent of bus company. [7]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

4 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = ct(1+t)^5,$$

where c is a constant.

(a) Find the value of c. [1]

(b) Find the value of $E(X)$. [2]

DO NOT WRITE IN THIS MARGIN

The random variable Y is the sum of two independent values of X.

(c) Write down the probability generating function of Y and hence find Var(Y). [4]

(d) Find $P(Y = 5)$. [2]

DO NOT WRITE IN THIS MARGIN

5 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 2, \\ \dfrac{(x-2)^2}{12} & 2 \leqslant x < 4, \\ 1 - \dfrac{(8-x)^2}{24} & 4 \leqslant x \leqslant 8, \\ 1 & x > 8. \end{cases}$$

(a) Sketch the graph of the probability density function of X. [3]

(b) Find $\mathrm{E}(X)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

(c) Find the exact value of the interquartile range of X. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

6 Seva is investigating the lengths of the tails of adult wallabies in two regions of Australia, X and Y. He chooses a random sample of 50 adult wallabies from region X and records the lengths, x cm, of their tails. He also chooses a random sample of 40 adult wallabies from region Y and records the lengths, y cm, of their tails. His results are summarised as follows.

$$\Sigma x = 1080 \qquad \Sigma x^2 = 23\,480 \qquad \Sigma y = 940 \qquad \Sigma y^2 = 22\,220$$

It cannot be assumed that the population variances of the two distributions are the same.

(a) Find a 90% confidence interval for the difference between the population mean lengths of the tails of adult wallabies in regions X and Y. [6]

DO NOT WRITE IN THIS MARGIN

The population mean lengths of the tails of adult wallabies in regions X and Y are μ_X cm and μ_Y cm respectively.

(b) Test, at the 10% significance level, the null hypothesis $\mu_Y - \mu_X = 1.1$ against the alternative hypothesis $\mu_Y - \mu_X > 1.1$. State your conclusion in the context of the question. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.