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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

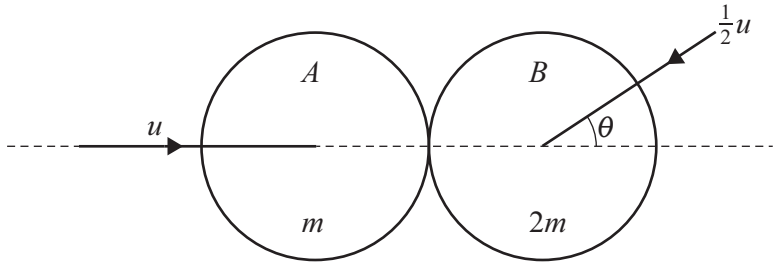
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1



Two smooth uniform spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $\frac{1}{2}u$ respectively. Immediately before the collision, A's direction of motion is along the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram).

As a result of the collision, the direction of motion of A is reversed and its speed is reduced to $\frac{1}{4}u$. The direction of motion of B again makes an angle θ with the line of centres, but on the opposite side of the line of centres. The speed of B is unchanged.

Find the value of the coefficient of restitution between the spheres. [4]

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2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $2mg$. A particle Q of mass km is attached to the other end of the string. Particle P lies on a smooth horizontal table. The string has part of its length in contact with the table and then passes through a small smooth hole H in the table.

Particle P moves in a horizontal circle on the surface of the table with constant speed $\sqrt{\frac{1}{2}ga}$. Particle Q hangs in equilibrium vertically below the hole with $HQ = \frac{1}{4}a$.

(a) Find, in terms of a, the extension in the string. [4]

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(b) Find the value of k. [2]

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3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When the particle is hanging vertically below O, it is projected horizontally with speed u so that it begins to move along a circular path. When P is at the lowest point of its motion, the tension in the string is T. When OP makes an angle θ with the upward vertical, the tension in the string is S.

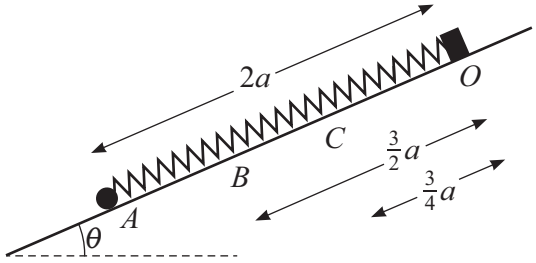
(a) Show that $S = T - 3mg(1 + \cos\theta)$. [5]

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(b) Given that $u = \sqrt{4ag}$, find the value of $\cos\theta$ when the string goes slack. [2]

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A light spring of natural length a and modulus of elasticity kmg is attached to a fixed point O on a smooth plane inclined to the horizontal at an angle θ, where $\sin\theta = \frac{3}{4}$. A particle of mass m is attached to the lower end of the spring and is held at the point A on the plane, where $OA = 2a$ and OA is along a line of greatest slope of the plane (see diagram).

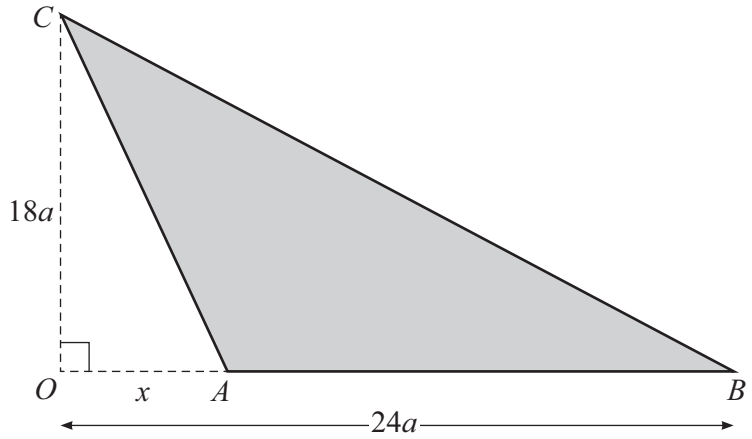
The particle is released from rest and is moving with speed V when it passes through the point B on the plane, where $OB = \frac{3}{2}a$. The speed of the particle is $\frac{1}{2}V$ when it passes through the point C on the plane, where $OC = \frac{3}{4}a$.

Find the value of k. [7]

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A uniform lamina is in the form of a triangle OBC, with $OC = 18a$, $OB = 24a$ and angle $COB = 90°$. The point A on OB is such that $OA = x$ (see diagram). The triangle OAC is removed from the lamina.

(a) Find, in terms of a and x, the distance of the centre of mass of the remaining object ABC from OC. [3]

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The object ABC is suspended from C. In its equilibrium position, the side AB makes an angle θ with the vertical, where $\tan\theta = \frac{6}{5}$.

(b) Find x in terms of a. [4]

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6 A particle P is projected with speed u m s^{-1} at an angle θ above the horizontal from a point O and moves freely under gravity. After 5 seconds the speed of P is $\frac{3}{4}u$.

(a) Show that $\frac{7}{16}u^2 - 100u\sin\theta + 2500 = 0$. [3]

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(b) It is given that the velocity of P after 5 seconds is perpendicular to the initial velocity.

Find, in either order, the value of u and the value of $\sin\theta$. [5]

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7 A parachutist of mass m kg opens his parachute when he is moving vertically downwards with a speed of $50\,\text{m}\,\text{s}^{-1}$. At time t s after opening his parachute, he has fallen a distance x m from the point where he opened his parachute, and his speed is $v\,\text{m}\,\text{s}^{-1}$. The forces acting on him are his weight and a resistive force of magnitude mv N.

(a) Find an expression for v in terms of t. [6]

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(b) Find an expression for x in terms of t. [3]

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(c) Find the distance that the parachutist has fallen, since opening his parachute, when his speed is $15\,m\,s^{-1}$. [2]

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