Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

5548612351

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

© UCLES 2023

1 The lengths of the leaves of a particular type of tree are normally distributed with mean μ cm. The lengths, x cm, of a random sample of 12 leaves of this type are recorded. The results are summarised as follows.

$$\sum x = 91.2 \qquad \sum x^2 = 695.8$$

Find a 95% confidence interval for μ. [4]

2 The children at two large schools, P and Q, are all given the same puzzle to solve. A random sample of size 10 is taken from the children at school P. Their individual times to complete the puzzle give a sample mean of 9.12 minutes and an unbiased variance estimate of 2.16 minutes^2. A random sample of size 12 is taken from the children at school Q. Their individual times, x minutes, to complete the puzzle are summarised by

$$\sum x = 99.6 \qquad \sum (x - \overline{x})^2 = 21.5,$$

where $\overline{x}$ is the sample mean. Times to complete the puzzle are assumed to be normally distributed with the same population variance.

Test at the 5% significance level whether the population mean time taken to complete the puzzle by children at school P is greater than the population mean time taken to complete the puzzle by children at school Q. [8]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 A random sample of 50 values of the continuous random variable X was taken. These values are summarised in the following table.

Interval	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$	$3 \leqslant x < 3.5$	$3.5 \leqslant x \leqslant 4$
Observed frequency	3	3	8	11	13	12

It is required to test the goodness of fit of the distribution with probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{24}\left(\dfrac{4}{x^2} + x^2\right) & 1 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The expected frequencies, correct to 4 decimal places, are given in the following table.

Interval	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$	$3 \leqslant x < 3.5$	$3.5 \leqslant x \leqslant 4$
Expected frequency	4.4271	a	6.1285	8.4549	b	14.9678

(a) Show that $a = 4.6007$ and find the value of b. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

(b) Carry out a goodness of fit test, at the 10% significance level, to test whether f is a satisfactory model for the data. [6]

4 A random sample of 13 technology companies is chosen and the numbers of employees in 2018 and in 2022 are recorded.

Company	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*	*M*
Number in 2018	104	19	126	234	970	514	35	149	429	12	86	304	1104
Number in 2022	106	24	127	228	1012	525	32	156	449	24	78	294	1154

A researcher claims that there has been an increase in the median number of employees at technology companies between 2018 and 2022.

(a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports this claim. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The researcher notices that the figures for company *G* have been recorded incorrectly. In fact, the number of employees in 2018 was 32 and the number of employees in 2022 was 35.

(b) Explain, with numerical justification, whether or not the conclusion of the test in part **(a)** remains the same. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 Harry has three coins.

- One coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{3}$.
- The second coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{4}$.
- The third coin is biased so that, when it is thrown, the probability of obtaining a head is $\frac{1}{5}$.

The random variable X is the number of heads that Harry obtains when he throws all three coins together.

(a) Find the probability generating function of X. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Isaac has two fair coins. The random variable Y is the number of heads that Isaac obtains when he throws both of his coins together. The random variable Z is the total number of heads obtained when Harry throws his three coins and Isaac throws his two coins.

(b) Find the probability generating function of Z, expressing your answer as a polynomial in t. [4]

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

(c) Use the probability generating function of Z to find $\mathrm{E}(Z)$. [2]

6 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{28}\left(\mathrm{e}^{\frac{1}{2}x} + 4\mathrm{e}^{-\frac{1}{2}x}\right) & 0 \leqslant x \leqslant 2\ln 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The random variable Y is defined by $Y = \mathrm{e}^{\frac{1}{2}(X)}$.

(b) Find the probability density function of Y. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the 30th percentile of Y. [3]

(d) Find $\mathrm{E}(Y^4)$. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2023