Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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7327543339

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x-y plane. The coordinates of the vertices are $O(0, 0)$, $A(15, 0)$, $B(9, 4)$ and $C(3, 4)$.

Find the x-coordinate of the centre of mass of the lamina. [4]

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

3 A particle P is projected with speed $25\,\text{m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2 s the speed of P is $15\,\text{m s}^{-1}$.

(a) Find the value of $\sin\theta$. [5]

(b) Find the range of the flight. [3]

4 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string and is held with the string taut at the point A. At A the string makes an angle θ with the upward vertical through O. The particle is projected perpendicular to the string in a downward direction from A with a speed u. It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O, the speed of the particle is $2u$ and the magnitude of the tension in the string is 10 times its magnitude at A.

It is given that $u = \sqrt{\frac{2}{3}ga}$.

(a) Find, in terms of m and g, the magnitude of the tension in the string at A. [6]

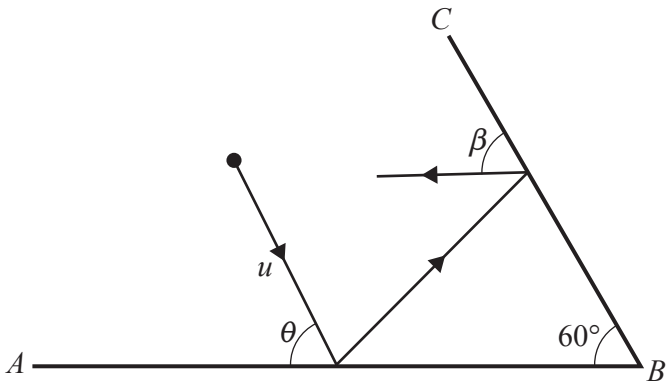
(b) Find the value of $\cos\alpha$. [2]

5 A particle P of mass 4 kg is moving in a horizontal straight line. At time t s the velocity of P is v m s^{-1} and the displacement of P from a fixed point O on the line is x m. The only force acting on P is a resistive force of magnitude $(4\mathrm{e}^{-x} + 12)\mathrm{e}^{-x}$ N. When $t = 0$, $x = 0$ and $v = 4$.

(a) Show by integration that $v = \dfrac{1 + 3\,\mathrm{e}^{x}}{\mathrm{e}^{x}}$. [4]

(b) Find an expression for x in terms of t. [4]

6



AB and *BC* are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass *m* is moving with speed *u* on the surface. The particle strikes *AB* at an angle θ with *AB*. It then strikes *BC* and rebounds at an angle β with *BC* (see diagram). The coefficient of restitution between the particle and each barrier is *e* and $\tan\theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of *e*. [4]

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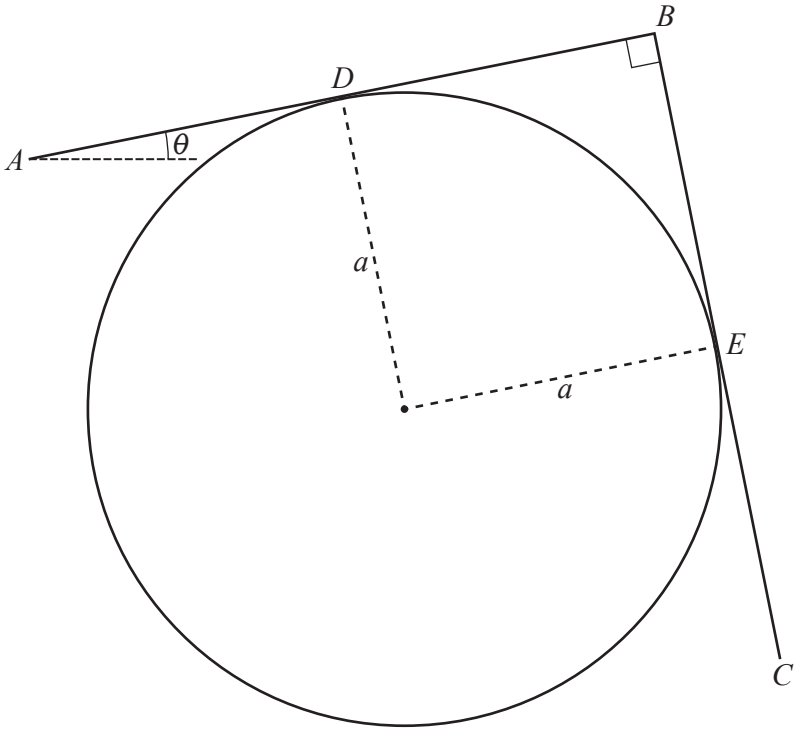
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(b) Find the size of angle β. [4]

7



A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. [6]

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(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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