Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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2144572331

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A random sample of 7 observations of a variable X are as follows.

8.26 7.78 7.92 8.04 8.27 7.95 8.34

The population mean of X is μ.

(a) Test, at the 10% significance level, the null hypothesis $\mu = 8.22$ against the alternative hypothesis $\mu < 8.22$. [6]

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(b) State an assumption necessary for the test in part **(a)** to be valid. [1]

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2 A driving school employs four instructors to prepare people for their driving test. The allocation of people to instructors is random. For each of the instructors, the following table gives the number of people who passed and the number who failed their driving test last year.

	Instructor *A*	Instructor *B*	Instructor *C*	Instructor *D*	Total
Pass	72	42	52	68	234
Fail	33	34	41	58	166
Total	105	76	93	126	400

Test at the 10% significance level whether success in the driving test is independent of the instructor. [7]

3 The continuous random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0, \\ \frac{1}{81}x^2 & 0 \leqslant x \leqslant 9, \\ 1 & x > 9. \end{cases}$$

(a) Find $\mathrm{E}\left(\sqrt{X}\right)$. [3]

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(b) Find $\mathrm{Var}\left(\sqrt{X}\right)$. [2]

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(c) The random variable Y is given by $Y^3 = X$. Find the probability density function of Y. [3]

4 A scientist is investigating the lengths of the leaves of birch trees in different regions. He takes a random sample of 50 leaves from birch trees in region A and a random sample of 60 leaves from birch trees in region B. He records their lengths in cm, x and y, respectively. His results are summarised as follows.

$$\Sigma x = 282 \qquad \Sigma x^2 = 1596 \qquad \Sigma y = 328 \qquad \Sigma y^2 = 1808$$

The population mean lengths of leaves from birch trees in regions A and B are μ_A cm and μ_B cm respectively.

Carry out a test at the 5% significance level to test the null hypothesis $\mu_A = \mu_B$ against the alternative hypothesis $\mu_A \neq \mu_B$. [8]

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5 Georgio has designed two new uniforms X and Y for the employees of an airline company. A random sample of 11 employees are each asked to assess each of the two uniforms for practicality and appearance, and to give a total score out of 100. The scores are given in the table.

Employee	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*
Uniform X	82	74	42	59	60	73	94	98	62	36	50
Uniform Y	78	75	63	56	67	82	99	90	72	48	61

(a) Give a reason why a Wilcoxon signed-rank test may be more appropriate than a t-test for investigating whether there is any evidence of a preference for one of the uniforms. [1]

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(b) Carry out a Wilcoxon matched-pairs signed-rank test at the 10% significance level. [7]

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[Turn over

6 Tanji has a bag containing 4 red balls and 2 blue balls. He selects 3 balls at random from the bag, without replacement. The number of red balls selected by Tanji is denoted by X.

(a) Find the probability generating function $G_X(t)$ of X. [2]

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Tanji also has two coins, each biased so that the probability of obtaining a head when it is thrown is $\frac{1}{4}$. He throws the two coins at the same time. The number of heads obtained is denoted by Y.

(b) Find the probability generating function $G_Y(t)$ of Y. [2]

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The random variable Z is the sum of the number of red balls selected by Tanji and the number of heads obtained.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

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(d) Use the probability generating function of Z to find $\mathrm{E}(Z)$ and $\mathrm{Var}(Z)$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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