Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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*7978932017*

**FURTHER MATHEMATICS** **9231/32**

Paper 3 Further Mechanics **May/June 2021**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.


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**1** A particle $P$ of mass 1 kg is moving along a straight line against a resistive force of magnitude $\frac{10\sqrt{v}}{(t+1)^2}$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of $P$ at time $t$ s. When $t = 0$, $v = 25$.

Find an expression for $v$ in terms of $t$. [5]

**2** A hollow hemispherical bowl of radius $a$ has a smooth inner surface and is fixed with its axis vertical. A particle $P$ of mass $m$ moves in horizontal circles on the inner surface of the bowl, at a height $x$ above the lowest point of the bowl. The speed of $P$ is $\sqrt{\frac{8}{3}ga}$.

Find $x$ in terms of $a$. [6]

**3** One end of a light elastic string, of natural length $a$ and modulus of elasticity $kmg$, is attached to a fixed point $A$. The other end of the string is attached to a particle $P$ of mass $4m$. The particle $P$ hangs in equilibrium a distance $x$ vertically below $A$.

**(a)** Show that $k = \dfrac{4a}{x-a}$. [1]

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An additional particle, of mass $2m$, is now attached to $P$ and the combined particle is released from rest at the original equilibrium position of $P$. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

**(b)** Find $x$ in terms of $a$. [6]

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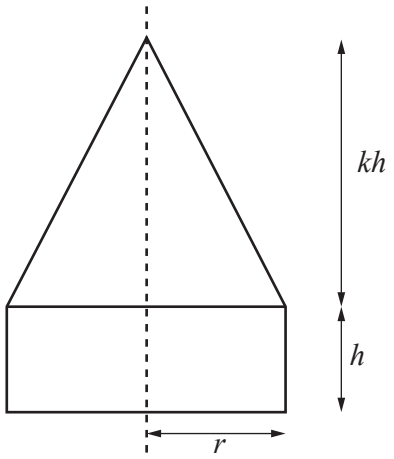
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**4**




A uniform solid circular cone has vertical height $kh$ and radius $r$. A uniform solid cylinder has height $h$ and radius $r$. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.

**(a)** Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\frac{h(k^2+4k+6)}{4(3+k)}$. [4]

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The solid is placed on a plane that is inclined to the horizontal at an angle $\theta$. The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan\theta = \frac{4}{3}$.

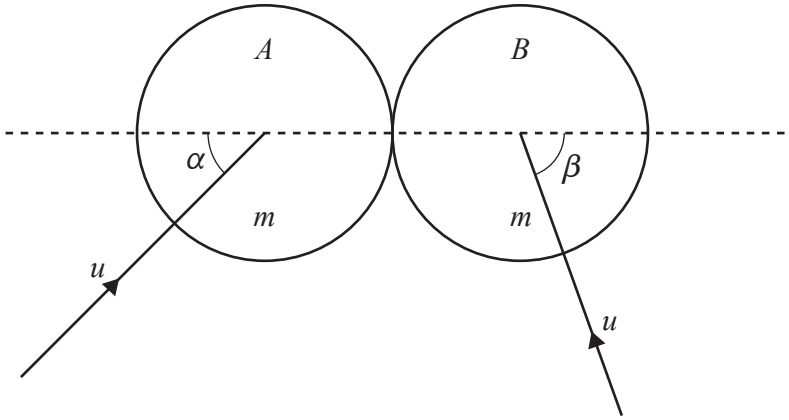
**(b)** Find the value of $k$. [3]

**5** A particle $P$ of mass $m$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle completes vertical circles with centre $O$. The points $A$ and $B$ are on the path of $P$, both on the same side of the vertical through $O$. $OA$ makes an angle $\theta$ with the downward vertical through $O$ and $OB$ makes an angle $\theta$ with the upward vertical through $O$.

The speed of $P$ when it is at $A$ is $u$ and the speed of $P$ when it is at $B$ is $\sqrt{ag}$. The tensions in the string at $A$ and $B$ are $T_A$ and $T_B$ respectively. It is given that $T_A = 7T_B$.

Find the value of $\theta$ and find an expression for $u$ in terms of $a$ and $g$. [8]

**6**




Two uniform smooth spheres $A$ and $B$ of equal radii each have mass $m$. The two spheres are each moving with speed $u$ on a horizontal surface when they collide. Immediately before the collision, $A$'s direction of motion makes an angle $\alpha$ with the line of centres, and $B$'s direction of motion makes an angle $\beta$ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

**(a)** Show that the direction of motion of $A$ after the collision is perpendicular to the line of centres. [4]

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The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

**(b)** Find the value of $\alpha$. [4]

**7** A particle $P$ is projected from a point $O$ on a horizontal plane and moves freely under gravity. The initial velocity of $P$ is $100\,\text{m}\,\text{s}^{-1}$ at an angle $\theta$ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which $P$'s height above the plane is $H$ m differ by 10 s.

**(a)** Find the value of $H$. [5]

**(b)** Find the magnitude and direction of the velocity of $P$ one second before it strikes the plane. [4]

## Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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