Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2833408233

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

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1 Young children are learning to read using two different reading schemes, *A* and *B*. The standards achieved are measured against the national average standard achieved and classified as above average, average or below average. For two randomly chosen groups of young children, the numbers in each category are shown in the table.

	Standard achieved		
	Above average	Average	Below average
Scheme *A*	31	35	22
Scheme *B*	19	50	43

Test at the 5% significance level whether standard achieved is independent of the reading scheme used. [6]

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2 A random sample of 40 observations of a random variable X and a random sample of 50 observations of a random variable Y are taken. The resulting values for the sample means, $\overline{x}$ and $\overline{y}$, and the unbiased estimates, s_x^2 and s_y^2, for the population variances are as follows.

$$\overline{x} = 24.4 \qquad \overline{y} = 17.2 \qquad s_x^2 = 10.2 \qquad s_y^2 = 11.1$$

Find a 90% confidence interval for the difference between the population means of X and Y. [5]

3 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{5}x & 0 \leqslant x < 2, \\ \frac{2}{15}(5-x) & 2 \leqslant x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

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(b) Find the median value of X. [2]

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(c) Find $\mathrm{E}(X^2)$. [2]

(d) Find $\mathrm{P}(1 \leqslant X \leqslant 3)$. [2]

4 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = 0.2t + 0.5t^2 + 0.3t^3.$$

The random variable Y is the sum of two independent observations of X.

(a) Find the probability generating function of Y, giving your answer as an expanded polynomial in t. [3]

(b) Use the probability generating function of Y to find $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$. [5]

5 Students at two colleges, A and B, are competing in a computer games challenge.

(a) The time taken for a randomly chosen student from college A to complete the challenge has a normal distribution with mean μ minutes. The times taken, x minutes, are recorded for a random sample of 10 students chosen from college A. The results are summarised as follows.

$$\Sigma x = 828 \qquad \Sigma x^2 = 68622$$

A test is carried out on the data at the 5% significance level and the result supports the claim that $\mu > k$.

Find the greatest possible value of k. [4]

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(b) A random sample of 8 students is chosen from college B. Their times to complete the same challenge give a sample mean of 79.8 minutes and an unbiased variance estimate of 9.966 minutes^2.

Use a 2-sample test at the 5% significance level to test whether the mean time for students at college B to complete the challenge is the same as the mean time for students at college A to complete the challenge. You should assume that the two distributions are normal and have the same population variance. [7]

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[Turn over

6 A biologist is studying the effect of nutrients on the heights to which plants grow. A random sample of 24 similar young plants is divided into two equal groups *A* and *B*. The plants in group *A* are fed with nutrients and water and the plants in group *B* are given only water. After four weeks, the height, in cm, of each plant is measured and the results are as follows.

Group *A*	12.3	11.8	12.1	13.2	11.1	10.6	13.8	12.0	12.2	12.4	13.5	13.9
Group *B*	11.7	10.8	10.9	11.3	11.2	12.6	11.0	10.5	11.9	12.5	10.7	11.6

The biologist decides to carry out a test at the 5% significance level to test whether the nutrients have resulted in an increase in growth.

(a) She carries out a Wilcoxon rank-sum test. Give a reason why this is an appropriate choice of test. [1]

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(b) Carry out the Wilcoxon rank-sum test for these results. [10]

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Additional Page

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