Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

5850054762

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

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1 Two randomly selected groups of students, with similar ranges of abilities, take the same examination in different rooms. One group of 140 students takes the examination with background music playing. The other group of 210 students takes the examination in silence. Each student is awarded a grade for their performance in the examination and the numbers from each group gaining each grade are shown in the following table.

	Grade awarded		
	A	B	C
Background music	49	51	40
Silence	93	68	49

Test at the 10% significance level whether grades awarded are independent of whether background music is playing during the examination. [6]

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2 The times, in milliseconds, taken by a computer to perform a certain task were recorded on 10 randomly chosen occasions. The times were as follows.

6.44 6.16 5.62 5.82 6.51 6.62 6.19 6.42 6.34 6.28

It is claimed that the median time to complete the task is 6.4 milliseconds.

(a) Carry out a Wilcoxon signed-rank test at the 5% significance level to test this claim. [6]

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(b) State an underlying assumption that is made when using a Wilcoxon signed-rank test. [1]

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3 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{16}\left(2 - \sqrt{x}\right) & 0 \leqslant x < 1, \\ \dfrac{3}{16\sqrt{x}} & 1 \leqslant x \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find E(X). [3]

The random variable Y is such that $Y = \sqrt{X}$.

(b) Find the probability density function of Y. [5]

4 A company has two different machines, X and Y, each of which fills empty cups with coffee. The manager is investigating the volumes of coffee, x and y, measured in appropriate units, in the cups filled by machines X and Y respectively. She chooses a random sample of 50 cups filled by machine X and a random sample of 40 cups filled by machine Y. The volumes are summarised as follows.

$$\Sigma x = 15.2 \quad \Sigma x^2 = 5.1 \quad \Sigma y = 13.4 \quad \Sigma y^2 = 4.8$$

The manager claims that there is no difference between the mean volume of coffee in cups filled by machine X and the mean volume of coffee in cups filled by machine Y.

Test the manager's claim at the 10% significance level. [9]

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5 A large number of children are competing in a throwing competition. The distances, in metres, thrown by a random sample of 8 children are as follows.

19.8 22.1 24.4 21.5 20.8 26.3 23.7 25.0

(a) Assuming that distances are normally distributed, test, at the 5% significance level, whether the population mean distance thrown is more than 22.0 metres. [7]

(b) Find a 95% confidence interval for the population mean distance thrown. [3]

6 A bag contains 4 red balls and 6 blue balls. Rassa selects two balls at random, without replacement, from the bag. The number of red balls selected by Rassa is denoted by X.

(a) Find the probability generating function, $G_X(t)$, of X. [2]

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Rassa also tosses two coins. One coin is biased so that the probability of a head is $\frac{2}{3}$. The other coin is biased so that the probability of a head is p. The probability generating function of Y, the number of heads obtained by Rassa, is $G_Y(t)$. The coefficient of t in $G_Y(t)$ is $\frac{7}{12}$.

(b) Find $G_Y(t)$. [3]

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The random variable Z is the sum of the number of red balls selected and the number of heads obtained by Rassa.

(c) Find the probability generating function of Z, expressing your answer as a polynomial. [3]

(d) Use the probability generating function of Z to find E(Z). [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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