Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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Question	Answer	Marks	Guidance
1(i)	$-\sin y\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=-(\sin y)^{-1}$	**M1 A1**	Differentiates implicitly once.
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=(\sin y)^{-2}\cos y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=-\cot y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	**M1 A1**	Differentiates again, AG.
		4	
1(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\left(\sin\dfrac{\pi}{3}\right)^{-1}=-\dfrac{2}{\sqrt{3}}$	**B1**	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=-\dfrac{1}{\sqrt{3}}\left(-\dfrac{2}{\sqrt{3}}\right)^2=-\dfrac{4}{3\sqrt{3}}=-\dfrac{4}{9}\sqrt{3}$	**B1**	AEF, must be exact.
		2	

Question	Answer	Marks	Guidance
2(i)	$\dfrac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1}=\dfrac{2(\cos(n-1)-\cos n)}{2\cos n\cos(n-1)}$	**M1**	Uses formulae for $\cos P \pm \cos Q$.
	$\dfrac{1}{\cos n}-\dfrac{1}{\cos(n-1)}$	**A1**	AG
		2	
2(ii)	$\displaystyle\sum_{n=1}^{N}\frac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1}=\sum_{n=1}^{N}\frac{1}{\cos n}-\frac{1}{\cos(n-1)}$ $=\dfrac{1}{\cos 1}-\dfrac{1}{1}+\dfrac{1}{\cos 2}-\dfrac{1}{\cos 1}+\ldots+\dfrac{1}{\cos N}-\dfrac{1}{\cos(N-1)}$	**M1**	Applies (i), shows enough terms and cancelation.
	$-1+\dfrac{1}{\cos N}$	**A1**	
		2	
2(iii)	$\cos N$ oscillates as $N\to\infty$ so $u_1+u_2+u_3+\ldots$ does not converge.	**B1**	States "oscillates" or refers to diverging values of $\cos N$.
		1	

Question	Answer	Marks	Guidance
3	$\overrightarrow{OP}=\begin{pmatrix}6+\lambda\\2+\lambda\\7\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}4\\4-6\mu\\\mu\end{pmatrix}\Rightarrow\overrightarrow{PQ}=\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}$	**M1 A1**	Finds $\overrightarrow{PQ}$.
	$\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=0$ **Or** $\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}=k\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&0\\0&-6&1\end{vmatrix}=k\begin{pmatrix}1\\-1\\-6\end{pmatrix}$	**M1**	Uses that dot product of $\overrightarrow{PQ}$ with line directions is 0. Or, alternatively, $\overrightarrow{PQ}$ is multiple of common perpendicular.
	$-2\lambda-6\mu=0$	**A1**	Deduces one equation. CWO.
	$\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}\begin{pmatrix}0\\-6\\1\end{pmatrix}=0\Rightarrow 6\lambda+37\mu=19$	**A1**	Deduces second equation. CWO.
	$\lambda=-3,\ \mu=1$	**M1 A1**	Solves simultaneous equations.
	$\overrightarrow{OP}=\begin{pmatrix}3\\-1\\7\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}4\\-2\\1\end{pmatrix}$	**A1**	States $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.
		8	

Question	Answer	Marks	Guidance
4(i)	$\int_0^1 x^2 e^{x^3}\ \mathrm{d}x = \frac{1}{3}\left[e^{x^3}\right]_0^1 = \frac{1}{3}(e-1)$	**M1 A1**	Must show working, AG.
		2	
4(ii)	$I_n = \int_0^1 x^{n-2} x^2 e^{x^3}\ \mathrm{d}x = \left[\frac{1}{3}x^{n-2}e^{x^3}\right]_0^1 - \frac{n-2}{3}\int_0^1 x^{n-3}e^{x^3}\ \mathrm{d}x$	**M1 A1**	Integrates by parts.
	$\frac{e}{3} - \frac{n-2}{3}I_{n-3} \Rightarrow 3I_n = e-(n-2)I_{n-3}$	**A1**	AG
		3	
4(iii)	$I_5 = \frac{1}{3}(e-3I_2) = \frac{1}{3}(e-e+1) = \frac{1}{3}$	**M1 A1**	Applies reduction formula once and uses **(i)**.
	$I_8 = \frac{1}{3}(e-6I_5) = \frac{1}{3}(e-2).$	**A1**	Must be exact.
		3	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{-2\left(\mathrm{e}^{t}-\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}$	**B1**	
	$\dfrac{\mathrm{d}y}{\mathrm{d}t}=\dfrac{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}-\left(\mathrm{e}^{t}-\mathrm{e}^{-t}\right)^{2}}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}=\dfrac{\left(2\mathrm{e}^{t}\right)\left(2\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}=\dfrac{4}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}$	**B1**	Differentiates and simplifies. Accept $1-\left(\dfrac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{\mathrm{e}^{t}+\mathrm{e}^{-t}}\right)^{2}$.
	$\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^{2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^{2}=\dfrac{4\left(\mathrm{e}^{t}-\mathrm{e}^{-t}\right)^{2}+16}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{4}}=\dfrac{4\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{4}}$	**M1 A1**	Attempt at writing $\left(\dfrac{\mathrm{d}x}{\mathrm{d}t}\right)^{2}+\left(\dfrac{\mathrm{d}y}{\mathrm{d}t}\right)^{2}$ as a square.
	$S=2\pi\displaystyle\int_{0}^{1}\left(\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{\mathrm{e}^{t}+\mathrm{e}^{-t}}\right)\left(\frac{2}{\mathrm{e}^{t}+\mathrm{e}^{-t}}\right)\mathrm{d}t=4\pi\int_{0}^{1}\frac{\mathrm{e}^{t}-\mathrm{e}^{-t}}{\left(\mathrm{e}^{t}+\mathrm{e}^{-t}\right)^{2}}\mathrm{d}t$	**A1**	Uses correct formula, simplifies to AG.
		5	
5(ii)	$S=4\pi\displaystyle\int_{2}^{e+e^{-1}}u^{-2}\,\mathrm{d}u=4\pi\left[-u^{-1}\right]_{2}^{\mathrm{e}+\mathrm{e}^{-1}}$	**M1 A1**	Applies given substitution.
	$=4\pi\left(\dfrac{1}{2}-\dfrac{1}{\mathrm{e}+\mathrm{e}^{-1}}\right).$	**A1**	AEF, must be exact.
		3	

Question	Answer	Marks	Guidance
6(i)	$y=(y+1)^3$	**M1**	Obtains an equation in y not involving radicals.
	$y=y^3+3y^2+3y+1\Rightarrow y^3+3y^2+2y+1=0$	**A1**	AG
	$S_3=-3$	**B1**	
		3	
6(ii)	$S_{-3}=\dfrac{\alpha^3\beta^3+\beta^3\gamma^3+\alpha^3\gamma^3}{\alpha^3\beta^3\gamma^3}=\dfrac{2}{-1}=-2$	**M1 A1**	
		2	
6(iii)	$S_6=(-3)^2-2(2)=5$	**M1 A1**	Uses $(\Sigma\alpha)^2=\Sigma\alpha^2+2\sum_{\alpha\neq\beta}\alpha\beta$. AG.
	$S_9=-3S_6-2S_3-3=-3(5)-2(-3)-3=-12$	**M1 A1**	Sums $y^3-3y^2+2y-1=0$ for $y=\alpha^3,\beta^3,\gamma^3$.
		4	

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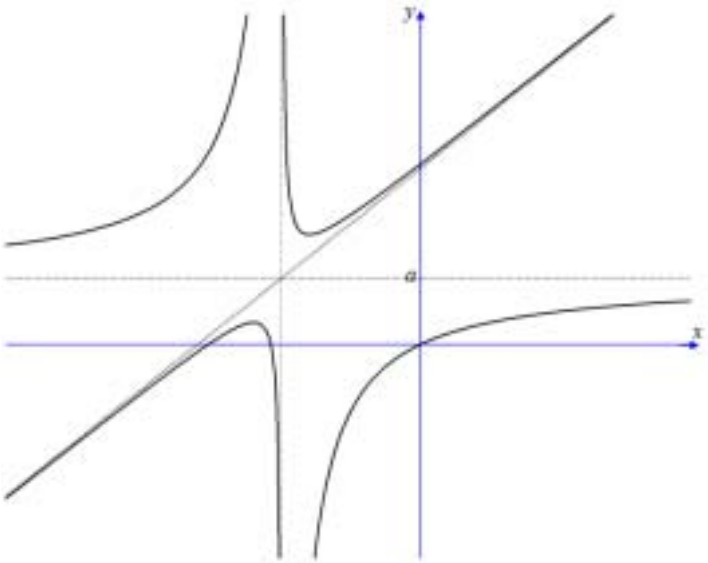
Question	Answer	Marks	Guidance
7	$10u^2+3u-1=0\Rightarrow(2u+1)(5u-1)=0$	**M1**	Axillary equation
	CF: $x=A\mathrm{e}^{-\frac{1}{2}t}+B\mathrm{e}^{\frac{1}{5}t}$	**A1**	
	PI: $x=p+qt\Rightarrow\dot{x}=q\Rightarrow\ddot{x}=0$	**M1**	Forms PI and differentiates.
	$3q-p-qt=t+2\Rightarrow q=-1, p=-5.$	**M1 A1**	Substitutes.
	GS: $x=A\mathrm{e}^{-\frac{1}{2}t}+B\mathrm{e}^{\frac{1}{5}t}-t-5$	**A1**	States general solution.
	$\dot{x}=-\frac{1}{2}A\mathrm{e}^{-\frac{1}{2}t}+\frac{1}{5}B\mathrm{e}^{\frac{1}{5}t}-1$	**M1**	Differentiates.
	$A+B=5$ $-\frac{1}{2}A+\frac{1}{5}B=1$	**M1**	Forms simultaneous equations.
	$A=0,\ B=5$	**A1**	
	$x=5\mathrm{e}^{\frac{1}{5}t}-t-5$	**A1**	States PS.
		10	

Question	Answer	Marks	Guidance
8(i)	$1=\frac{z^1-1}{z-1}$ So true when $n=1$.	**B1**	Shows base case.
	Assume that $1+z+\ldots+z^{k-1}=\frac{z^k-1}{z-1}$	**B1**	States inductive hypothesis.
	Then $1+z+\ldots+z^{k-1}+z^k=\frac{z^k-1}{z-1}+z^k=\frac{z^k-1+z^k(z-1)}{z-1}=\frac{z^{k+1}-1}{z-1}$, so true when $n=k+1$	**M1 A1**	Combines fractions.
	$H_k \rightarrow H_{k+1}$ Hence, by induction, true for all positive integers.	**A1**	States conclusion.
		5	

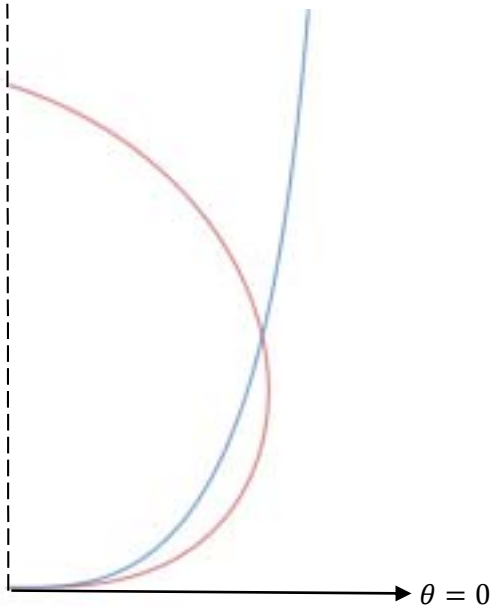
Question	Answer	Marks	Guidance
8(ii)	Since $\lvert z\rvert<1$, $\sum_{m=0}^{\infty} z^m = \frac{-1}{z-1}$	**B1**	States $\lvert z\rvert<1$ and uses formula for sum to infinity of geometric progression.
	$\sum_{m=1}^{\infty} 2^{-m}\sin m\theta = \text{Im}\left(\sum_{m=0}^{\infty} z^m\right) = \text{Im}\left(\frac{-1}{\frac{1}{2}\cos\theta + \text{i}\frac{1}{2}\sin\theta - 1}\right)$	**M1 A1**	Uses de Moivre's theorem.
	$\text{Im}\left(\frac{-\left(\frac{1}{2}\cos\theta - 1 - \text{i}\frac{1}{2}\sin\theta\right)}{\frac{1}{4}\cos^2\theta - \cos\theta + 1 + \frac{1}{4}\sin^2\theta}\right)$	**M1**	Multiply numerator and denominator by conjugate.
	$\frac{\frac{1}{2}\sin\theta}{\frac{5}{4}-\cos\theta} = \frac{2\sin\theta}{5-4\cos\theta}$	**A1**	States imaginary part, AG.
		5	

Question	Answer	Marks	Guidance
9(i)	$\mathbf{A}^2\mathbf{e} = \mathbf{A}(\mathbf{Ae}) = \lambda\mathbf{Ae} = \lambda^2\mathbf{e}$	**M1 A1**	AG
		2	
9(ii)	Eigenvalues of $\mathbf{A}$ are $n, 2n$ and $3n$.	**B1**	
	$\lambda = n$: $\mathbf{e_1} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 0 & n & 0 \end{vmatrix} = \begin{pmatrix} -3n \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1 A1**	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 2n$: $\mathbf{e_2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -n & 1 & 3 \\ 0 & 0 & n \end{vmatrix} = \begin{pmatrix} n \\ n^2 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ n \\ 0 \end{pmatrix}$	**A1**	
	$\lambda = 3n$: $\mathbf{e_3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2n & 1 & 3 \\ 0 & -n & 0 \end{vmatrix} = \begin{pmatrix} 3n \\ 0 \\ 2n^2 \end{pmatrix} = t\begin{pmatrix} 3 \\ 0 \\ 2n \end{pmatrix}$	**A1**	
	Eigenvalues of $\mathbf{A} + n\mathbf{I}$ are $2n, 3n$ and $4n$	**B1**	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & n & 0 \\ 0 & 0 & 2n \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} (2n)^2 & 0 & 0 \\ 0 & (3n)^2 & 0 \\ 0 & 0 & (4n)^2 \end{pmatrix}$	**M1 A1**	Or correctly matched permutations of columns.
		8	

Question	Answer	Marks	Guidance
10(i)	$x=-5$ and $y=a$	**B1 B1**	
		2	
10(ii)	$x^2+(a+10)x+5a+26=(x+5)(x+a+5)+1$	**M1**	By inspection or long division.
	oblique asymptote is $y=x+a+5$	**A1**	
		2	
10(iii)	$x^2+10x+5a+26=0$	**M1**	Puts y–values equal and forms quadratic equation.
	$10^2-4(5a+26)=-4-20a<0$ so no intersection point	**A1**	Correct discriminant and conclusion.
		2	
10(iv)	$(x+5)(2x+a+10)-x^2-ax-10x-5a-26=0$	**M1**	Differentiates and forms quadratic equation.
	$x^2+10x+24=0$	**A1**	
	Stationary points are $(-4,a+2)$ and $(-6,a-2)$	**A1**	Must have both points.
		3	

Question	Answer	Marks	Guidance
10(v)		**B1**	Asymptotes drawn, intersection correct.
		B1	C_1 correct.
		B1	C_2 correct.
		3	

Question	Answer	Marks	Guidance
11E(i)	$x = \sqrt{2}\theta^{\frac{1}{2}}\cos\theta$	**M1**	Uses $x = r\cos\theta$.
	$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sqrt{2}\theta^{\frac{1}{2}}\cos\theta\right) = \sqrt{2}\left(-\theta^{\frac{1}{2}}\sin\theta + \frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta\right) = 0$	**M1 A1**	Sets derivative of $r\cos\theta$ equal to zero.
	$-\theta^{\frac{1}{2}}\sin\theta + \frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta = 0 \Rightarrow \cos\theta = 2\theta\sin\theta \Rightarrow 2\theta\tan\theta = 1$	**A1**	AG
	$2(0.6)\tan(0.6) - 1 = -0.179$ and $2(0.7)\tan(0.7) - 1 = 0.179$	**B1**	Shows sign change.
		5	

Question	Answer	Marks	Guidance
11E(ii)	$2\theta = \theta\sec^2\theta \Rightarrow \theta^{\frac{1}{2}}\left(\sec\theta - \sqrt{2}\right) = 0 \Rightarrow \theta = \frac{\pi}{4}$	**M1 A1**	Finds value of θ.
		2	
11E(iii)		**B1**	Correct shape.
		B1	Intersection correct.
		2	

Question	Answer	Marks	Guidance
11E(iv)	$\frac{1}{2}\int_0^{\frac{\pi}{4}} 2\theta \,\mathrm{d}\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \theta\sec^2\theta \,\mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} \theta\left(2-\sec^2\theta\right)\mathrm{d}\theta$	**M1**	Forms correct integral.
	$\frac{1}{2}\left[\theta\left(2\theta-\tan\theta\right)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(2\theta-\tan\theta\right)\mathrm{d}\theta$	**M1 A1**	Integrates by parts.
	$\frac{1}{2}\left[\theta\left(2\theta-\tan\theta\right)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[\theta^2+\ln\cos\theta\right]_0^{\frac{\pi}{4}}$	**A1**	
	$\frac{\pi}{8}\left(\frac{\pi}{2}-1\right) - \frac{1}{2}\left(\left(\frac{\pi}{4}\right)^2 + \frac{1}{2}\ln 2\right) = \frac{1}{4}\ln 2 + \frac{\pi}{8}\left(\frac{\pi}{4}-1\right).$	**A1**	AEF, must be exact.
		5	
11O(i)(a)	$\begin{pmatrix} -1 & 2 & 3 & 4 \\ 1 & 0 & 1 & -1 \\ 1 & -2 & -3 & a \\ 1 & 2 & 5 & 2 \end{pmatrix} \to\cdots\to \begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & a+4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\quad v_1 \quad v_2 \quad v_3 \quad v_4 \qquad\qquad v_1' \quad v_2' \quad v_3' \quad v_4'$	**M1A1**	Reduces **M** or $\mathbf{M}^{\mathrm{T}}$ to echelon form.
	$\dim V = \text{rank} = 3$.	**A1**	
	$c_1v_1' + c_2v_2' + c_3v_4' = 0 \Rightarrow c_1 = c_2 = c_3 = 0$	**M1**	Shows v_1', v_2', v_4' are linearly independent.
	Thus v_1, v_2, v_4 are linearly independent (and so form a basis for V).	**A1**	
		5	

Question	Answer	Marks	Guidance
11O(i)(b)	$\begin{pmatrix} -1 & 2 & 4 & x \\ 1 & 0 & -1 & y \\ 1 & -2 & a & z \\ 1 & 2 & 2 & t \end{pmatrix} \to \cdots \to \begin{pmatrix} -1 & 2 & 4 & x \\ 0 & 2 & 3 & y+x \\ 0 & 0 & a+4 & z+x \\ 0 & 4 & 6 & t+x \end{pmatrix}$	**M1 A1**	$x = -\alpha + 2\beta + 4\gamma$ $y = \alpha - \gamma$ Uses row operations or $z = \alpha - 2\beta + a\gamma$ $t = \alpha + 2\beta + 2\gamma$
	System is consistent when $t + x = 2(y + x) \Rightarrow x + 2y = t$ Or $(-\alpha + 2\beta + 4\gamma) + 2(\alpha - \gamma) = \alpha + 2\beta + 2\gamma = t$	**M1 A1**	AG
		4	
11O(ii)	$-x + 2y + 3z + 4t = 0$ $2y + 4z + 3t = 0$	**M1**	Finds basis for null space.
	$t = \mu, \quad z = \lambda, \quad y = -2\lambda - \frac{3}{2}\mu, \quad x = -\lambda + \mu$	**A1**	
	A basis is $\left\{ \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 2 \end{pmatrix} \right\} = \{\mathbf{e}_1, \mathbf{e}_2\}$	**A1**	AEF
	$\mathbf{M}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ so particular solution is $\mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	**B1**	Finds particular solution.
	General solution is $\mathbf{x} = \mathbf{e} + \lambda\mathbf{e}_1 + \mu\mathbf{e}_2$	**A1**	**FT** Accept *their* basis. Must have correct particular solution.
		5	

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