Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

4173383933

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

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1 A particle P is moving in a fixed circle of radius 0.8 m. At time t s its velocity is $(t^2 - t + 2)\,\text{m s}^{-1}$. Find the magnitudes of the radial and the transverse components of the acceleration of P when $t = 2$. [3]

Radial component .....

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Transverse component .....

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2 Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e.

(i) Show that after the collision A moves with speed $\frac{1}{5}u(4 - e)$ and find the speed of B. [4]

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Sphere B continues to move until it collides with a fixed smooth vertical barrier which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

(ii) Find the value of e. [3]

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The spheres A and B now collide directly again.

(iii) Determine whether sphere B collides with the barrier for a second time. [2]

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3 A particle P moves on the positive x-axis in simple harmonic motion. The centre of the motion is a distance d m from the origin O, where $0 < d < 6.5$. The points A and B are on the positive x-axis, with $OA = 6.5$ m and $OB = 7.5$ m. The magnitude of the acceleration of P when it is at B is twice the magnitude of the acceleration of P when it is at A.

(i) Find d. [2]

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The period of the motion is π s and the maximum acceleration of P during the motion is $10\,\text{m}\,\text{s}^{-2}$.

(ii) Find the speed of P when it is 7 m from O. [5]

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(iii) Find the time taken by P to travel directly from A to B. [3]

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4 A uniform rod AB has length $2a$ and weight W. The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The angle between the rod and the horizontal is θ, where $\tan\theta = \frac{4}{3}$. One end of a light inextensible rope is attached to a point C on the rod. The other end is attached to a point where the vertical wall and the horizontal ground meet. The rope is taut and perpendicular to the rod. The rope and rod are in a vertical plane perpendicular to the wall.

(i) Show that $AC = \frac{18}{25}a$. [2]

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The magnitude of the frictional force at A is equal to one quarter of the magnitude of the normal reaction force at A.

(ii) Show that the tension in the rope is $\frac{1}{4}W$. [6]

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(iii) Find expressions, in terms of W, for the magnitudes of the normal reaction forces at A and B. [2]

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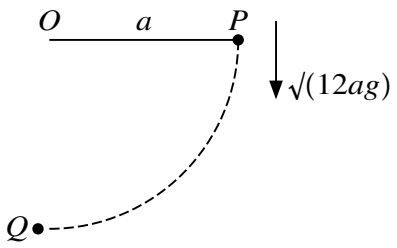
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A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held with the string taut and horizontal. It is projected downwards with speed $\sqrt{(12ag)}$. At the lowest point of its motion, P collides directly with a particle Q of mass km which is at rest (see diagram). In the collision, P and Q coalesce. The tension in the string immediately after the collision is half of its value immediately before the collision. Find the possible values of k. [11]

6 A random sample of 15 observations of pairs of values of two variables gives a product moment correlation coefficient of 0.430.

(i) Test at the 10% significance level whether there is evidence of non-zero correlation between the variables. [4]

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A second random sample of N observations gives a product moment correlation coefficient of 0.615. Using a 5% significance level, there is evidence of positive correlation between the variables.

(ii) Find the least possible value of N, justifying your answer. [2]

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7 The probability that a driver passes an advanced driving test has a fixed value p for each attempt. A driver keeps taking the test until he passes. The random variable X denotes the number of attempts required for the driver to pass. The variance of X is 3.75.

(i) Show that $15p^2 + 4p - 4 = 0$ and hence find the value of p. [4]

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(ii) Find $P(X = 5)$. [1]

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(iii) Find $P(3 \leqslant X \leqslant 7)$. [2]

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8 For a random sample of 6 observations of pairs of values (x, y), the equation of the regression line of y on x is $y = bx + 1.306$, where b is a constant. The corresponding equation of the regression line of x on y is $x = 0.6331y + d$, where d is a constant. The values of x from the sample are

$$2.3 \quad 2.8 \quad 3.7 \quad p \quad 6.1 \quad 6.4$$

and the sum of the values of y is 46.5. The product moment correlation coefficient is 0.9797.

(i) Find the value of b correct to 3 decimal places. [2]

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(ii) Find the value of p. [4]

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(iii) Use the equation of the regression line of x on y to estimate the value of x when $y = 8.5$. [3]

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9 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{20}\left(3 - \frac{1}{\sqrt{x}}\right) & 1 \leqslant x \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = \sqrt{X}$.

(i) Show that the probability density function of Y is given by

$$\mathrm{g}(y) = \begin{cases} \frac{1}{10}(3y - 1) & 1 \leqslant y \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$ [7]

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(ii) Find the mean value of Y. [2]

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10 During the summer months, all members of a large swimming club take part in intensive training. The times taken to swim 50 metres at the beginning of the summer and at the end of the summer are recorded for each member of the club. The time taken, in seconds, at the beginning of the summer is denoted by x and the time taken at the end of the summer is denoted by y. For a random sample of 9 members the results are shown in the following table.

Member	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
x	38.5	40.2	32.3	35.1	36.2	41.4	32.0	38.2	38.2
y	37.4	38.1	31.6	34.7	34.2	38.6	31.8	36.3	36.8

The swimming coach believes that, on average, the time taken by a swimmer to swim 50 metres will decrease by more than one second as a result of the intensive training.

(i) Stating suitable hypotheses and assuming a normal distribution, test the coach's belief at the 10% significance level. [8]

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(ii) Find a 95% confidence interval for the population mean time taken to swim 50 metres after the intensive training, assuming a normal distribution. [4]

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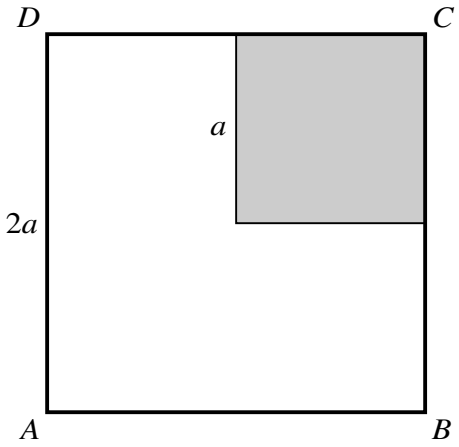
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11 Answer only **one** of the following two alternatives.

EITHER



An object is formed from a square frame $ABCD$ with a square lamina attached in one corner of the frame. The frame consists of four identical thin rods, each of mass M and length $2a$. The lamina has mass kM and edges of length a. It has one vertex at C and adjacent sides in contact with CB and CD (see diagram).

(i) Show that the moment of inertia of the object about an axis l through A perpendicular to the plane of the object is $\frac{2}{3}Ma^2(7k + 20)$. [8]

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The object is released from rest with the edge AB horizontal and D vertically above A. The object rotates freely about the fixed axis l. The angular speed of the object is $\frac{1}{2}\sqrt{\left(\frac{5g}{a}\right)}$ when D is first vertically below A.

(ii) Find the value of k. [6]

OR

A scientist carries out an experiment to investigate the quantity X, which takes the values 0, 1, 2, 3, 4, 5 or 6. He believes that the values taken by X follow a binomial distribution. He conducts 250 trials. His results are summarised in the following table.

x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0

(i) Show that unbiased estimates of the mean and variance for these results are 1.876 and 1.266 respectively, correct to 3 decimal places. By evaluating the mean and variance of the distribution B(6, 0.313), explain why X could have this distribution. [4]

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The expected frequencies corresponding to the distribution B(6, 0.313) are shown in the following table.

x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0
Expected frequency	26.3	71.9	81.8	49.7	17.0	3.1	0.2

(ii) Show how the expected frequency for $x = 4$ is calculated. [2]

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(iii) Test at the 5% significance level whether the scientist's belief is correct. [8]

Additional Page

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