Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

0710483476

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

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1 A bullet of mass m kg is fired horizontally into a fixed vertical block of material. It enters the block horizontally with speed $250\,\text{m s}^{-1}$ and emerges horizontally with speed $70\,\text{m s}^{-1}$ after 0.04 s. The block offers a constant horizontal resisting force of magnitude 450 N. Find the value of m. [3]

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2 A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O. The points A and B are on the line, on opposite sides of O, with $OA = 1.6$ m and $OB = 1.2$ m. The ratio of the speed of P at A to its speed at B is 3 : 4.

(i) Find the amplitude of the motion. [4]

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The maximum speed of P during its motion is $\frac{1}{3}\pi$ m s^{-1}.

(ii) Find the period of the motion. [2]

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(iii) Find the time taken for P to travel directly from A to B. [3]

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3 Two identical uniform small spheres A and B, each of mass m, are moving towards each other in a straight line on a smooth horizontal surface. Their speeds are u and ku respectively, and they collide directly. The coefficient of restitution between the spheres is e. Sphere B is brought to rest by the collision.

(i) Show that $e = \dfrac{k-1}{k+1}$. [3]

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(ii) Given that 60% of the total initial kinetic energy is lost in the collision, find the values of k and e. [6]

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4 A uniform rod AB has length $2a$ and weight W. The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The rod is in a vertical plane that is perpendicular to the wall. The angle between the rod and the horizontal is θ. A particle of weight $5W$ hangs from the rod at the point C, with $AC = xa$, where $0 < x < 1$.

(i) By taking moments about A, show that the magnitude of the normal reaction at B is $\dfrac{W(5x + 1)}{2 \tan \theta}$. [3]

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The particle of weight $5W$ is now moved a distance a up the rod, so that $AC = (x + 1)a$. This results in the magnitude of the normal reaction at B being double its previous value. The system remains in equilibrium with the rod at angle θ with the horizontal.

(ii) Show that $x = \frac{4}{5}$. [3]

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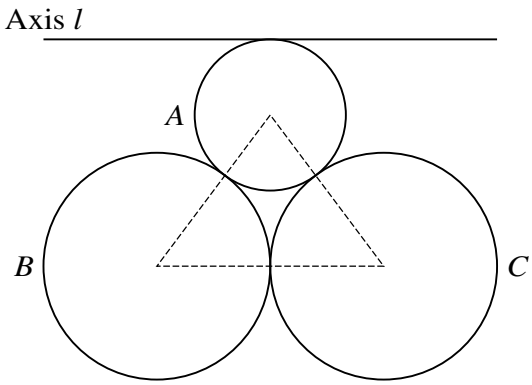
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The coefficient of friction between the rod and the ground is $\frac{2}{3}$.

(iii) Given that the rod is about to slip when the particle of weight $5W$ is in its second position, find the value of $\tan\theta$. [5]

5



Three thin uniform rings A, B and C are joined together, so that each ring is in contact with each of the other two rings. Ring A has radius $2a$ and mass $3M$; rings B and C each have radius $3a$ and mass $2M$. The rings lie in the same plane and the centres of the rings are at the vertices of an isosceles triangle. The object consisting of the three rings is free to rotate about the horizontal axis l which is tangential to ring A, in the plane of the object and perpendicular to the line of symmetry of the object (see diagram).

(i) Show that the moment of inertia of the object about the axis l is $180Ma^2$. [7]

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(ii) Show that small oscillations of the object about the axis l are approximately simple harmonic, and state the period. [5]

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6 The continuous random variable X has distribution function given by

$$\mathrm{F}(x) = \begin{cases} 1 - \mathrm{e}^{-0.4x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $\mathrm{P}(X > 2)$. [2]

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(ii) Find the interquartile range of X. [4]

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7 A large number of athletes are taking part in a competition. The masses, in kg, of a random sample of 7 athletes are as follows.

98.1 105.0 92.2 89.8 99.9 95.4 101.2

Assuming that masses are normally distributed, test, at the 10% significance level, whether the mean mass of athletes in this competition is equal to 94 kg. [7]

8 A manufacturer produces three types of car: hatchbacks, saloons and estates. Each type of car is available in one of three colours: silver, blue and red. The manufacturer wants to know whether the popularity of the colour of the car is related to the type of car. A random sample of 300 cars chosen by customers gives the information summarised in the following table.

		Colour of car		
		Silver	Blue	Red
Type of car	Hatchback	53	36	41
	Saloon	29	40	31
	Estate	28	24	18

Test at the 10% significance level whether the colour of car chosen by customers is independent of the type of car. [8]

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9 At a ski resort, the probability of snow on any particular day is constant and equal to p. The skiing season begins on 1 November. The random variable X denotes the day of the skiing season on which the first snowfall occurs. (For example, if the first snowfall is on 5 November, then $X = 5$.) The variance of X is $\frac{4}{9}$.

(i) Show that $4p^2 + 9p - 9 = 0$ and hence find the value of p. [4]

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(ii) Find the probability that the first snowfall will be on 3 November. [1]

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(iii) Find the probability that the first snowfall will not be before 4 November. [2]

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(iv) Find the least integer N so that the probability of the first snowfall being on or before the Nth day of November is more than 0.999. [4]

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10 The times taken to run 400 metres by students at two large colleges P and Q are being compared. There is no evidence that the population variances are equal. The time taken by a student at college P and the time taken by a student at college Q are denoted by x seconds and y seconds respectively. A random sample of 50 students from college P and a random sample of 60 students from college Q give the following summarised data.

$$\Sigma x = 2620 \qquad \Sigma x^2 = 138\,200 \qquad \Sigma y = 3060 \qquad \Sigma y^2 = 157\,000$$

(i) Using a 10% significance level, test whether, on average, students from college P take longer to run 400 metres than students from college Q. [9]

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(ii) Find a 90% confidence interval for the difference in the mean times taken to run 400 metres by students from colleges P and Q. [3]

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11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held so that the string is taut, with OP horizontal. The particle is projected downwards with speed $\sqrt{\left(\frac{2}{5}ag\right)}$ and begins to move in a vertical circle. The string breaks when its tension is equal to $\frac{11}{5}mg$.

(i) Show that the string breaks when OP makes an angle θ with the downward vertical through O, where $\cos\theta = \frac{3}{5}$. Find the speed of P at this instant. [6]

(ii) For the subsequent motion after the string breaks, find the distance OP when the particle P is vertically below O. [6]

OR

The regression line of y on x, obtained from a random sample of 6 pairs of values of x and y, has equation

$$y = 0.25x + k,$$

where k is a constant. The values from the sample are shown in the following table.

x	4	5	7	8	10	14
y	5	8	p	7	p	9

(i) Find the value of p and the value of k. [6]

(ii) Find the product moment correlation coefficient for the data. [2]

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(iii) Test, at the 5% significance level, whether there is evidence of positive correlation between the variables. [4]

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Additional Page

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