Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2018**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	$a_R = (2^2 – 2 + 2)^2 / 0{\cdot}8 = 4^2 / 0{\cdot}8 = 20$ [m s^{-2}]	**M1 A1**	Find radial acceleration a_R at $t = 2$ from v^2/r
	$a_T = 2t – 1 = 3$ [m s^{-2}]	**B1**	Find transverse acceleration a_T at $t = 2$ by differentiation
		3	

Question	Answer	Marks	Guidance
2(i)	$4mv_A + mv_B = 4mu$ (AEF)	**M1**	Use momentum (allow m omitted)
	$v_B – v_A = eu$	**M1**	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = (1/5)\,(4 – e)\,u$ **AG**	**A1**	Combine to verify/find speeds of A and B after collision
	$v_B = (4/5)\,(1 + e)\,u$	**A1**	
		4	
2(ii)	$v_B' = [–]\,¾\,e\,v_B$ [$= –\,(3/5)\,e\,(1 + e)\,u$]	**M1**	Relate vel. v_B' of B after colln. with wall to v_B
	$(1/5)\,(4 – e)\,u = (3/5)\,e\,(1 + e)\,u$	**M1**	Equate speeds of A and B (ignore sign of v_B' for both M1s)
	$3e^2 + 4e – 4 = 0$, $e = 2/3$	**A1**	Solve resulting quadratic for e, [implicitly] rejecting root –2
		3	
2(iii)	*EITHER:* $4mw_A + mw_B = 3mv_A$ [$= 2mu$] (AEF) $w_B – w_A = e \times 2v_A = (4/3)v_A$ *or* $(8/9)\,u$ (AEF) [$w_A = ⅓\,v_A = (2/9)\,u$], $w_B = (5/3)v_A$ *or* $(10/9)\,u$ *OR:* Momentum before colln. is $3mv_A = 2mu > 0$, so after colln. momentum *or* speed of $B > 0$ $w_B > 0$ so B collides again with barrier	**M1 A1**	Use momentum (allow m omitted) with $v_A = –v_B'$ [$= ⅔\,u$] Use Newton's law (M0 if LHS signs inconsistent) Combine to find velocity of B after final collision Allow any similar valid argument Allow $v_A > 0$ by [implicit] inspection
		2	

Question	Answer	Marks	Guidance
3(i)	$\omega^2(7{\cdot}5 - d) = 2\ \omega^2(6{\cdot}5 - d)$, $d = 5{\cdot}5$	**M1 A1**	Find d by relating accelns. at A and B (ω^2 may be omitted)
		2	
3(ii)	$\omega = 2\pi/T = 2$	**B1**	Find ω from period T (may be implied)
	$a = 10/\omega^2 = 2{\cdot}5$ [m] (ignore sign; FT on ω)	**B1√**	Find amplitude a from max. acceleration 10
	$v^2 = \omega^2(a^2 - x^2)$, $x = 7 - d$ [= 1·5]	**M1, M1**	Find speed v when $OP = 7$ (2nd M1 dep. on 1st M1)
	$= 2^2(2{\cdot}5^2 - 1{\cdot}5^2) = 16$, $v = 4$ [m s^{-1}]	**A1**	
		5	
3(iii)	$\omega^{-1}\sin^{-1}(x_B/a) - \omega^{-1}\sin^{-1}(x_A/a)$ *or* $\omega^{-1}\cos^{-1}(x_A/a) - \omega^{-1}\cos^{-1}(x_B/a)$	**M1**	Find time from A to B from $x = a\sin\omega t$ or $a\cos\omega t$ (all terms must be correct (FT on a, ω) for M1)
	$= ½\sin^{-1}(2/2{\cdot}5) - ½\sin^{-1}(1/2{\cdot}5)$ *or* $½\cos^{-1}(1/2{\cdot}5) - ½\cos^{-1}(2/2{\cdot}5)$ $= ½(0{\cdot}9273 - 0{\cdot}4115)$ *or* $0{\cdot}4636 - 0{\cdot}2058$ *or* $½(1{\cdot}1593 - 0{\cdot}6435)$ *or* $0{\cdot}5796 - 0{\cdot}3218$	**A1**	To 3 d.p., AEF throughout
	= 0·258 [s]	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	$AC = AD \cos\theta$ and $AD = 2a \cos\theta$ [$= 6a/5$] so $AC = 2a \cos^2\theta = 2a\,(3/5)^2 = 18a/25$ **AG**	M1 A1	Find AC (D denotes other end of rope from C)
		2	
4(ii)	A: $R_B \times 2a \sin\theta - W \times a \cos\theta - T \times AC = 0$ [$R_B \times 8a/5 - W \times 3a/5 - T \times 18a/25 = 0$ so $40\,R_B - 15\,W - 18\,T = 0$] B: $F_A \times 2a \sin\theta - R_A \times 2a \cos\theta + W \times a \cos\theta$ $+ T \times (2a - AC) = 0$ [$F_A \times 8a/5 - R_A \times 6a/5 + W \times 3a/5 + T \times 32a/25 = 0$ so $15\,W + 32\,T = 30\,R_A - 40\,F_A = 20\,R_A$] C: $F_A \times AC \sin\theta - R_A \times AC \cos\theta + R_B \times (2a - AC) \sin\theta$ $- W \times (a - AC) \cos\theta = 0$ [$F_A \times 72a/125 - R_A \times 54a/125 + R_B \times 128a/125$ $- W \times 21a/125 = 0$ so $128\,R_B - 21\,W = 54\,R_A - 72\,F_A = 36\,R_A$] D: $R_A \times 2a \cos\theta - R_B \times 2a \sin\theta - W \times a \cos\theta = 0$ [$R_A \times 6a/5 - R_B \times 8a/5 - W \times 3a/5 = 0$ so $6\,R_A - 8\,R_B - 3W = 0$] G: $F_A \times a \sin\theta - R_A \times a \cos\theta + R_B \times a \sin\theta$ $+ T \times (a - AC) = 0$ [$F_A \times 4a/5 - R_A \times 3a/5 + R_B \times 4a/5 + T \times 7a/25 = 0$ so $20\,F_A - 15\,R_A + 20\,R_B + 7\,T = 0$]	M1 A1	Take moments for rod about one chosen point (Note that a vertical resolution will then give T, earning 6/6) (G is mid-point of AB)
	Horizontally: $R_B - F_A = T \sin\theta$ [$= 4T/5$]	B1	Find two more indep. eqns, e.g. resolution of forces on rod
	Vertically: $R_A - W = T \cos\theta$ [$= 3T/5$]	B1	(a second moment eqn. may be used)
	$T = W/4$ **AG**	M1 A1	Find or verify T using $F_A = ¼\,R_A$, $\sin\theta = 4/5$, $\cos\theta = 3/5$
		6	

Question	Answer	Marks	Guidance
4(iii)	$R_A = 23W / 20$ *or* $1{\cdot}15\ W$ [$F_A = 23W / 80 = 0{\cdot}2875\ W$]	**B1**	Find R_A, R_B (can assume $T = W/4$)
	$R_B = 39W / 80$ *or* $0{\cdot}487_{[5]}\ W$	**B1**	
		2	

Question	Answer	Marks	Guidance
5	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mga$	**M1**	Find v_1^2 at lowest point from consvn. of energy (M0 if no m)
	$v_1^2 = (12 + 2)\ ag = 14\ ag$	**A1**	
	$Mv_2 = mv_1$ with $M = (1 + k)\ m$	**M1**	Find new v_2 from consvn. of momentum
	$v_2 = v_1 / (1 + k)$ [$= \sqrt{(14\ ag)} / (1 + k)$]	**A1**	
	$T_1 = mv_1^2/a + mg = (14 + 1)\ mg = 15mg$	**M1 A1**	Find tension T_1 just before collision by using $F = ma$ radially
	$T_2 = Mv_2^2/a + Mg$ $= (1 + k)\ \{14 / (1 + k)^2 + 1\}\ mg$	**M1**	Find tension T_2 just after collision by using $F = ma$ radially (M1 needs M, not m, throughout)
	or $\{14 / (1 + k) + (1 + k)\}\ mg$ (AEF)	**A1**	
	$14 + (1 + k)^2 = 15\ (1 + k)/2$	**M1**	Equate T_2 and $\frac{1}{2}\ T_1$ to give any eqn in k
	$2k^2 - 11\ k + 15 = 0$, $k = 2{\cdot}5$ *or* 3 (M1 dep)	**M1 A1**	Solve resulting quadratic for k (M1 dep on all previous M1s, and requires *quadratic* eqn)
		11	

Question	Answer	Marks	Guidance
6(i)	H_0: $\rho = 0$, H_1: $\rho \neq 0$	**B1**	State both hypotheses (B0 for r …)
	EITHER: $r_{15,\,10\%} = 0{\cdot}441$	***B1**	State or use correct tabular two-tail r-value
	Accept H_0 if $0{\cdot}430 <$ tab. r-value (AEF)	**M1**	State or imply valid method for conclusion
	OR: $t_r = r\sqrt{((n-2) / (1 - r^2))} = 1{\cdot}72$, $t_{13,0{\cdot}95} = 1{\cdot}771$	**(*B1)**	(Rarely seen)
	Accept H_0 if $\lvert t_r \rvert <$ tab. T-value (AEF)	**(M1)**	
	No [non-zero] correlation (AEF)	**A1**	Correct conclusion (dep *B1)
		4	
6(ii)	$r_{8,\,5\%} = 0{\cdot}621$, $r_{9,\,5\%} = 0{\cdot}582$ so $N_{min} = 9$ (or equivalent argument)	**M1 A1**	Find N_{min} from relevant [one-tail] tabular values **SC:** Award B1 for stating 9 with inadequate justification
		2	

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Question	Answer	Marks	Guidance
7(i)	$(1-p)/p^2 = 3{\cdot}75$, $15p^2 + 4p - 4 = 0$ **AG**	**M1 A1**	Find given eqn. for p using $\text{Var}(X) = (1-p)/p^2$
	$(5p-2)(3p+2) = 0$, $p = 2/5$ *or* $0{\cdot}4$	**M1 A1**	Solve quadratic for p (A0 if $p = -⅔$ not [implicitly] rejected)
		4	
7(ii)	$\text{P}(X = 5) = (1-p)^4 p = 0{\cdot}6^4 \times 0{\cdot}4 = 0{\cdot}0518$ *or* 162/3125	**B1**	Find $\text{P}(X = 5)$
		1	
7(iii)	*EITHER:* $\text{P}(3 \leqslant X \leqslant 7) = (1-p)^2 - (1-p)^7$	**M1**	Find $\text{P}(3 \leqslant X \leqslant 7)$
	$= 0{\cdot}6^2 - 0{\cdot}6^7 = 0{\cdot}36 - 0{\cdot}028 = 0{\cdot}332$	**A1**	M0 for $\text{P}(X \leqslant 7) - \text{P}(X \leqslant 3)$ [= 0·188] or similar error
	OR: $\text{P}(3 \leqslant X \leqslant 7) = \sum_{i=3}^{7} (1-p)^{i-1} p$	**(M1)**	
	$= (0{\cdot}6^2 + 0{\cdot}6^3 + 0{\cdot}6^4 + 0{\cdot}6^5 + 0{\cdot}6^6) \times 0{\cdot}4$ $= 0{\cdot}830016 \times 0{\cdot}4 = 0{\cdot}332$	**(A1)**	
		2	

Question	Answer	Marks	Guidance
8(i)	$b \times 0{\cdot}6331 = 0{\cdot}9797^2$, $b = 1{\cdot}516$ (3 d.p.)	**M1 A1**	Find b from given gradients and coefficient
		2	
8(ii)	$46{\cdot}5/6$ [= 7·75] = $b \times \bar{x} + 1{\cdot}306$ *or* $b \times (\Sigma x)/6 + 1{\cdot}306$	**M1**	Find p from means and regression line of y on x
	$\bar{x} = 4{\cdot}25$ *or* $\Sigma x = 25{\cdot}5$ (3 s.f.)	**A1**	(intermediate values may be implied)
	$6\ \bar{x}$ *or* $\Sigma x = 21{\cdot}3 + p$, $p = 4{\cdot}2$ (1 d.p.)	**M1 A1**	
		4	
8(iii)	*EITHER:* $(\Sigma x)/6 = 0{\cdot}6331\ (\Sigma y)/6 + d$ [$d = -0{\cdot}656$]	**M1**	Find d from means and regression line of x on y
	$x = 0{\cdot}6331 \times 8{\cdot}5 + d = 4{\cdot}725_{[3]}$ *or* 4·73	**M1 A1**	Estimate x when $y = 8.5$ using regression line of x on y
	OR: $x - (\Sigma x)/6 = 0{\cdot}6331\ \{8{\cdot}5 - (\Sigma y)/6\}$,	**(M2)**	Combine above into single step
	$x = 4{\cdot}725_{[3]}$ *or* 4·73	**(A1)**	
	SC: $x = (8{\cdot}5 - 1{\cdot}306)/b$ [= 4·7452] = 4·75	**(B1)**	**SC:** Estimate x when $y = 8.5$ using regression line of y on x
		3	

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Question	Answer	Marks	Guidance
9(i)	*EITHER:* $F(x) = \int f(x)\,dx = (1/20)(3x - 2\sqrt{x}\ [+ c])$ $c = -1$ so $F(x) = (1/20)(3x - 2\sqrt{x} - 1)$	**M1**	Find or state distribution function $F(x)$ for $1 \leqslant x \leqslant 9$ (may be implied by $G(y)$)
	or $(3/20)x - (1/10)\sqrt{x} - 1/20$ $G(y)$ [$= P(Y < y) = P(\sqrt{X} < y) = P(X < y^2)$]	**A1**	Find or state $G(y)$ from $Y = \sqrt{X}$ for $1 \leqslant x \leqslant 9$ or $1 \leqslant y \leqslant 3$
	$= F(y^2)$ $= (1/20)(3y^2 - 2y - 1)$	**M1**	Allow A1√ as FT on expression found for $F(x)$
	or $(3/20)y^2 - (1/10)y - 1/20$	**A2**	Verify $g(y)$ (differentiation may be implied)
	$g(y) = G'(y) = (1/10)(3y - 1)$ **AG** [for $1 \leqslant y \leqslant 3$, $g(y) = 0$ otherwise]	**M1 A1**	**SC** Missing/incorrect c can earn M1 M1 A1√ M1 (max 4/7)
		7	
	OR: Use of $g(y) = f(x) \times \lvert dx / dy \rvert$	**(*M1)**	Reference to standard result required (not in syllabus)
	$f(x) = (1/20)(3 - 1/y)$ (dep *M1)	**(M1 A1)**	Find $f(x)$ using $x = y^2$
	$dx/dy = 2y$ (dep *M1)	**(M1 A1)**	Find dx/dy using $x = y^2$
	$g(y) = f(x) \times dx/dy = (1/10)(3y - 1)$ **AG** [for $1 \leqslant y \leqslant 3$, $g(y) = 0$ otherwise]	**(M1 A1)**	
		(7)	
9(ii)	$E(Y) = (1/10)\int(3y^2 - y)\,dy$	**M1**	Find mean of Y from $\int y\,g(y)\,dy$
	$= (1/10)[y^3 - \frac{1}{2}y^2]_1^3 = 11/5$ *or* 2·2	**A1**	
		2	

Question	Answer	Marks	Guidance
10(i)	H_0: $\mu_x - \mu_y = 1$, H_1: $\mu_x - \mu_y > 1$ (AEF)	**B1**	State both hypotheses (B0 for $\bar{x}$ …)
	d_i: 1·1 2·1 0·7 0·4 2·0 2·8 0·2 1·9 1·4	**M1**	Consider differences d_i, e.g. $x_i - y_i$
	$\bar{d}$ = 12·6 / 9 = 1·4	**B1**	Find sample mean
	$s^2 = (23{\cdot}72 - 12{\cdot}6^2/9) / 8$ [= 0·76 *or* $0{\cdot}8718^2$]	**M1**	Estimate population variance (allow biased here: [$0{\cdot}6756$ *or* $0{\cdot}8219^2$])
	$t_{8,\,0.9}$ = 1·397 *or* 1·40	**B1**	State or use correct tabular *t*-value
	$t = (\bar{d} - 1) / (s/\sqrt{9}) = 1{\cdot}38$ [Accept H_0:] No evidence for coach's belief	**M1 A1**	Find *t* (or compare $\bar{d} - 1 = 0{\cdot}4$ with $t_{8,\,0.9}\, s/\sqrt{9} = 0{\cdot}406$) Consistent conclusion (FT on both *t*-values)
	or time will not decrease by more than 1 s (AEF)	**B1√**	
			SC Wrong (hypothesis) test can earn only B1 for hypotheses B1√ for conclusion (max 2/8)
		8	
10(ii)	$s_y^2 = (11\,395{\cdot}79 - 319{\cdot}5^2/9) / 8$ = $6{\cdot}693$ *or* $2{\cdot}587^2$ (to 3 sf)	**B1**	Estimate population variance using *y*'s sample (allow use of biased here: $5{\cdot}950$ *or* $2{\cdot}439^2$)
	$319{\cdot}5/9 \pm t\sqrt{(s_y^2/9)}$	**M1**	Find confidence interval (M0 if s^2 not s_y^2)
	$t_{8,\,0.975}$ = 2·306 *or* 2·31	**A1**	State or use correct tabular value of *t*
	$35{\cdot}5 \pm 1{\cdot}99$ *or* [33·5, 37·5] (allow $35{\cdot}5 \pm 2{\cdot}0$)	**A1**	Evaluate C.I. (either form) **SC** B1 (max 1/4) for $1{\cdot}4 \pm 0{\cdot}67$ *or* [0·73, 2·07]
		4	

Question	Answer	Marks	Guidance
11A(i)	$I_{lamina} = \frac{1}{3}\, kM((\frac{1}{2}\, a)^2 + (\frac{1}{2}\, a)^2)$ $[= (k/6)\, Ma^2]$	**B1**	Find or state MI of lamina about axis at its centre
	$I'_{lamina} = I_{lamina} + kM((3a/2)^2 + (3a/2)^2)$ $[= (14k/3)\, Ma^2]$	**M1 A1**	Find MI of lamina about axis at A
	EITHER: $I_{AB} = \frac{1}{3}\, Ma^2 + Ma^2$ $[= (4/3)\, Ma^2]$	**B1**	Find or state MI of AB (or AD) about axis at A
	$I_{BC} = \frac{1}{3}\, Ma^2 + M((a^2 + (2a)^2)$ $[= (16/3)\, Ma^2]$	**M1 A1**	Find or state MI of BC (or DC) about axis at A
	OR: $I_{rod} = \frac{1}{3}\, Ma^2 + Ma^2$ $[= (4/3)\, Ma^2]$	**(B1)**	Find or state MI of any rod about axis at centre of frame
	$I_{frame} = 4 \times I_{rod} + 4M(a^2 + a^2)$ $[= (40/3)\, Ma^2]$	**(M1 A1)**	Find or state MI of frame about axis at A
	$I = (14k/3 + 2 \times 4/3 + 2 \times 16/3)\, Ma^2$ *or* $(14k/3 + 40/3)\, Ma^2$	**M1**	Verify MI of system about axis at A
	$= \frac{2}{3}\, Ma^2\,(7k + 20)$ **AG**	**A1**	A0 if inadequate explanation
		8	
11A(ii)	$\frac{1}{2}\, I\omega^2 = kMg \times 3a + 2Mg \times 2a + Mg \times 4a$ *or* $kMg \times 3a + 4Mg \times 2a$	**M1 A2**	Find ω^2 or angular speed ω when D below B by energy Award A1 if error in only term
	$\omega^2 = \{3(3k + 8) / (7k + 20)\}\, g/a$ $4 \times 3(3k + 8) = 5(7k + 20)$, $k = 4$	**A1**	Find k by equating ω^2 to $\{\frac{1}{2}\sqrt{(5g/a)}\}^2$
		M1 A1	
		6	

Question	Answer	Marks	Guidance
11B(i)	$\bar{x} = 469/250 = 1{\cdot}876$ *and* **AG** $\sigma^2 = (1195 - 469^2/250) / 249 = 1{\cdot}266$ **AG**	**B1**	Verify mean and unbiased variance of data, showing method B0 if biased variance, but allow other marks (max 3/4)
	$6 \times 0{\cdot}313 = 1{\cdot}878$ *and* $6 \times 0{\cdot}313 \times 0{\cdot}687 = 1{\cdot}29$[0]	**M1 A1**	Find mean *np and* variance *npq* of binomial distribution
	Means and variances are similar for X and B(6, 0·313)	**A1**	Valid explanation (AEF; needs 1·878, 1·290 correct to 0·002)
		4	
11B(ii)	$250 \times {}^6C_4 \times 0{\cdot}313^4 \times 0{\cdot}687^2$ [$= 250 \times 0{\cdot}06795 = 16{\cdot}99$] Allow $49{\cdot}7 \times ({}^6C_4 \times 0{\cdot}313) / ({}^6C_3 \times 0{\cdot}687)$	**M1 A1**	Show how expected frequency for $x = 4$ is found (AEF for ${}^6C_4 = 15$)
		2	
11B(iii)	H_0: [Binomial] distribution fits data (AEF)	**B1**	State (at least) null hypothesis
	O_i : 22 83 72 53 20 E_i : 26·3 71·9 81·8 49·7 20·3	***M1 A1**	Combine last 3 cells so that all exp. value ≥ 5
	$X^2 = 0{\cdot}703 + 1{\cdot}714 + 1{\cdot}174 + 0{\cdot}219 + 0{\cdot}004$ $= 3{\cdot}81$ (to 3 s.f.)	**M1 A1**	Find value of χ^2 from $\Sigma (E_i - O_i)^2 / E_i$ [or $\Sigma O_i^2/E_i - n$] (A1 dep *M1)
	No. n of cells: 7 6 5 $\chi^2_{n-2,\,0.95}$: 11·07 9·488 7·815	**B1√**	State or use consistent tabular value $\chi^2_{n-2,\,0.95}$ (to 3 s.f.) [FT on number, n, of cells used to find X^2]
	Accept H_0 if $X^2 <$ tabular value	**M1**	State or imply valid method for conclusion
	$3{\cdot}81$ [± 0·01] $< 7{\cdot}81$[5] so distribution fits *or* scientist's belief is correct	**A1**	Conclusion (AEF; requires both values approx. correct)
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