Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/22**

Paper 2 **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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1 A small smooth sphere $\boldsymbol{P}$ of mass $2\boldsymbol{m}$ is at rest on a smooth horizontal surface. A horizontal impulse of magnitude $8\boldsymbol{mu}$ is given to $\boldsymbol{P}$. Subsequently $\boldsymbol{P}$ collides directly with a fixed smooth vertical barrier at right angles to $\boldsymbol{P}$'s direction of motion. Given that the coefficient of restitution between $\boldsymbol{P}$ and the barrier is 0.75, find the speed of $\boldsymbol{P}$ after the collision. [2]

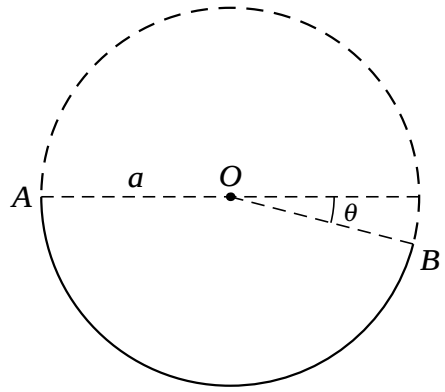
2 The point $\boldsymbol{O}$ is on the fixed line $\boldsymbol{l}$. Points $\boldsymbol{A}$ and $\boldsymbol{B}$ on $\boldsymbol{l}$ are such that $\boldsymbol{OA} = 0.5$ m and $\boldsymbol{OB} = 0.75$ m, with $\boldsymbol{A}$ between $\boldsymbol{O}$ and $\boldsymbol{B}$. A particle $\boldsymbol{P}$ of mass $\boldsymbol{m}$ oscillates on $\boldsymbol{l}$ in simple harmonic motion with centre $\boldsymbol{O}$. The ratio of the kinetic energy of $\boldsymbol{P}$ when it is at $\boldsymbol{A}$ to its kinetic energy when it is at $\boldsymbol{B}$ is 12 : 11. Find the amplitude of the motion. [3]

Given that the greatest speed of $\boldsymbol{P}$ is $0.6\,\mathrm{m\,s^{-1}}$, find the time taken by $\boldsymbol{P}$ to travel directly from $\boldsymbol{A}$ to $\boldsymbol{B}$. [5]

3 Three small smooth spheres $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ have equal radii and have masses $\boldsymbol{m}$, $9\boldsymbol{m}$ and $\boldsymbol{km}$ respectively. They are at rest on a smooth horizontal table and lie in a straight line with $\boldsymbol{B}$ between $\boldsymbol{A}$ and $\boldsymbol{C}$. The coefficient of restitution between any pair of the spheres is $\boldsymbol{e}$. Sphere $\boldsymbol{A}$ is projected directly towards $\boldsymbol{B}$ with speed $\boldsymbol{u}$. Given that half of the total kinetic energy is lost as result of the collision between $\boldsymbol{A}$ and $\boldsymbol{B}$, find the value of $\boldsymbol{e}$. [7]

After $\boldsymbol{B}$ and $\boldsymbol{C}$ collide they move in the same direction and the speed of $\boldsymbol{C}$ is twice the speed of $\boldsymbol{B}$. Find the value of $\boldsymbol{k}$. [3]

4



A smooth wire is in the form of an arc $\boldsymbol{AB}$ of a circle, of radius $\boldsymbol{a}$, that subtends an obtuse angle $\pi - \theta$ at the centre $\boldsymbol{O}$ of the circle. It is given that $\sin\theta = \frac{1}{4}$. The wire is fixed in a vertical plane, with $\boldsymbol{AO}$ horizontal and $\boldsymbol{B}$ below the level of $\boldsymbol{O}$ (see diagram). A small bead of mass $\boldsymbol{m}$ is threaded on the wire and projected vertically downwards from $\boldsymbol{A}$ with speed $\sqrt{(\frac{3}{10}\boldsymbol{ga})}$.

(i) Find the reaction between the bead and the wire when the bead is vertically below $\boldsymbol{O}$. [3]

(ii) Find the speed of the bead as it leaves the wire at $\boldsymbol{B}$. [3]

(iii) Show that the greatest height reached by the bead is $\frac{1}{8}\boldsymbol{a}$ above the level of $\boldsymbol{O}$. [4]

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5 A uniform rectangular thin sheet of glass $ABCD$, in which $AB = 8a$ and $BC = 6a$, has mass $\frac{3}{5}M$. Each of the edges AB, BC, CD and DA has a thin strip of metal attached to it, as a border to the glass. The strips along AB and CD each have mass M, and the strips along BC and DA each have mass $\frac{1}{3}M$. Show that the moment of inertia of the whole object (glass and metal strips) about an axis through A perpendicular to the plane of the object is $128Ma^2$. [8]

The object is free to rotate about this axis, which is fixed and smooth. The object hangs in equilibrium with C vertically below A. It is displaced through a small angle and released from rest. Show that it will move in approximate simple harmonic motion and state the period of the motion. [5]

6 A pair of coins is thrown repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X. State the expected value of X. [1]

Find the probability that

(i) exactly 4 throws are required to obtain a pair of heads, [2]

(ii) fewer than 6 throws are required to obtain a pair of heads. [2]

7 The random variable T is the lifetime, in hours, of a randomly chosen decorative light bulb of a particular type. It is given that T has a negative exponential distribution with mean 1000 hours.

(i) Write down the probability density function of T. [1]

(ii) Find the probability that a randomly chosen bulb of this type has a lifetime of more than 2000 hours. [3]

A display uses 10 randomly chosen bulbs of this type, and they are all switched on simultaneously. Find the greatest value of t such that the probability that they are all alight at time t hours is at least 0.9. [4]

8 Weekly expenses claimed by employees at two different branches, A and B, of a large company are being compared. Expenses claimed by an employee at branch A and by an employee at branch B are denoted by \$$x$ and \$$y$ respectively. A random sample of 60 employees from branch A and a random sample of 50 employees from branch B give the following summarised data.

$$\Sigma x = 6060 \qquad \Sigma x^2 = 626\,220 \qquad \Sigma y = 4750 \qquad \Sigma y^2 = 464\,500$$

Using a 2% significance level, test whether, on average, employees from branch A claim the same as employees from branch B. [9]

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[Turn over

9 A random sample of 200 observations of the continuous random variable $\boldsymbol{X}$ was taken and the values are summarised in the following table.

Interval	$1 \leqslant \boldsymbol{x} < 2$	$2 \leqslant \boldsymbol{x} < 3$	$3 \leqslant \boldsymbol{x} < 4$	$4 \leqslant \boldsymbol{x} < 5$	$5 \leqslant \boldsymbol{x} < 6$	$6 \leqslant \boldsymbol{x} < 7$	$7 \leqslant \boldsymbol{x} < 8$
Observed frequency	63	45	32	25	22	7	6

It is required to test the goodness of fit of the distribution with probability density function f given by

$$\mathrm{f}(\boldsymbol{x}) = \begin{cases} \dfrac{1}{\boldsymbol{x} \ln 8} & 1 \leqslant \boldsymbol{x} < 8, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$1 \leqslant \boldsymbol{x} < 2$	$2 \leqslant \boldsymbol{x} < 3$	$3 \leqslant \boldsymbol{x} < 4$	$4 \leqslant \boldsymbol{x} < 5$	$5 \leqslant \boldsymbol{x} < 6$	$6 \leqslant \boldsymbol{x} < 7$	$7 \leqslant \boldsymbol{x} < 8$
Expected frequency	66.67	$\boldsymbol{p}$	27.67	$\boldsymbol{q}$	17.54	14.83	12.84

Show that $\boldsymbol{p} = 39.00$, correct to 2 decimal places, and find the value of $\boldsymbol{q}$. [4]

Carry out a goodness of fit test at the 5% significance level. [6]

10 Samples of rock from a number of geological sites were analysed for the quantities of two types, $\boldsymbol{X}$ and $\boldsymbol{Y}$, of rare minerals. The results, in milligrams, for 10 randomly chosen samples, each of 10 kg, are summarised as follows.

$$\Sigma \boldsymbol{x} = 866 \qquad \Sigma \boldsymbol{x}^2 = 121\,276 \qquad \Sigma \boldsymbol{y} = 639 \qquad \Sigma \boldsymbol{y}^2 = 55\,991 \qquad \Sigma \boldsymbol{xy} = 73\,527$$

Find the product moment correlation coefficient. [4]

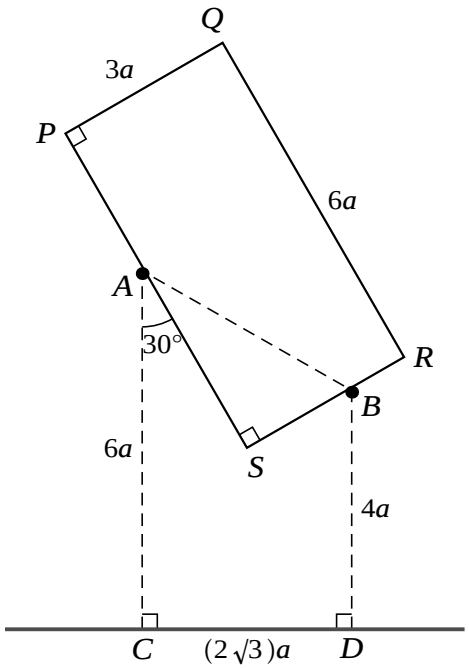
Stating your hypotheses, test at the 5% significance level whether there is non-zero correlation between quantities of the two rare minerals. [4]

Find the equation of the regression line of $\boldsymbol{x}$ on $\boldsymbol{y}$ in the form $\boldsymbol{x} = \boldsymbol{py} + \boldsymbol{q}$, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are constants to be determined. [3]

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11 Answer only **one** of the following two alternatives.

EITHER



The points $\boldsymbol{C}$ and $\boldsymbol{D}$ are at a distance $(2\sqrt{3})\boldsymbol{a}$ apart on a horizontal surface. A rough peg $\boldsymbol{A}$ is fixed at a vertical distance $6\boldsymbol{a}$ above $\boldsymbol{C}$ and a smooth peg $\boldsymbol{B}$ is fixed at a vertical distance $4\boldsymbol{a}$ above $\boldsymbol{D}$. A uniform rectangular frame $\boldsymbol{PQRS}$, with $\boldsymbol{PQ} = 3\boldsymbol{a}$ and $\boldsymbol{QR} = 6\boldsymbol{a}$, is made of rigid thin wire and has weight $\boldsymbol{W}$. It rests in equilibrium in a vertical plane with $\boldsymbol{PS}$ on $\boldsymbol{A}$ and $\boldsymbol{SR}$ on $\boldsymbol{B}$, and with angle $\boldsymbol{SAC} = 30°$ (see diagram).

(i) Show that $\boldsymbol{AB} = 4\boldsymbol{a}$ and that angle $\boldsymbol{SAB} = 30°$. [4]

(ii) Show that the normal reaction at $\boldsymbol{A}$ is $\frac{1}{2}\boldsymbol{W}$. [3]

(iii) Find the frictional force at $\boldsymbol{A}$. [7]

OR

The time taken for a randomly chosen student at College $\boldsymbol{P}$ to complete a particular puzzle has a normal distribution with mean μ minutes. The times, $\boldsymbol{x}$ minutes, are recorded for a random sample of 8 students chosen from the college. The results are summarised as follows.

$$\Sigma \boldsymbol{x} = 42.8 \qquad \Sigma \boldsymbol{x}^2 = 236.0$$

Find a 95% confidence interval for μ. [4]

A test is carried out on this sample data, at the 10% significance level. The test supports the claim that $\mu > \boldsymbol{k}$. Find the greatest possible value of $\boldsymbol{k}$. [3]

A random sample, of size 12, is taken from the students at College $\boldsymbol{Q}$. Their times to complete the puzzle give a sample mean of 4.60 minutes and an unbiased variance estimate of 1.962 minutes^2. Use a 2-sample test at the 10% significance level to test whether the mean time for students at College $\boldsymbol{Q}$ to complete the puzzle is less than the mean time for students at College $\boldsymbol{P}$ to complete the puzzle. You should state any assumptions necessary for the test to be valid. [7]

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