Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/12**

Paper 1 **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 2 9 8 7 0 6 9 9 1 4 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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1 The equation $x^3 + px + q = 0$, where p and q are constants, with $q \neq 0$, has one root which is the reciprocal of another root. Prove that $p + q^2 = 1$. [5]

2 Expand and simplify $(r + 1)^4 - r^4$. [1]

Use the method of differences together with the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n + 1)^2.$$ [4]

3 Prove by mathematical induction that, for all non-negative integers n,

$$11^{2n} + 25^n + 22$$

is divisible by 24. [6]

4 Obtain the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 6\frac{\mathrm{d}x}{\mathrm{d}t} + 25x = 195 \sin 2t.$$ [6]

5 The curve C has polar equation $r = a(1 + \sin\theta)$, where a is a positive constant and $0 \leqslant \theta < 2\pi$. Draw a sketch of C. [2]

Find the exact value of the area of the region enclosed by C and the half-lines $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$. [4]

6 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 & 3 \\ 2 & 0 & 0 & 5 \\ 6 & -2 & 2 & 11 \\ 10 & -3 & 3 & 19 \end{pmatrix}.$$

(i) Find the rank of $\mathbf{M}$ and state a basis for the range space of T. [4]

(ii) Obtain a basis for the null space of T. [4]

7 Use de Moivre's theorem to show that

$$\tan 5\theta = \frac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4},$$

where $t = \tan\theta$. [4]

Deduce that the roots of the equation $t^4 - 10t^2 + 5 = 0$ are $\pm\tan\frac{1}{5}\pi$ and $\pm\tan\frac{2}{5}\pi$. [3]

Hence show that $\tan\frac{1}{5}\pi\tan\frac{2}{5}\pi = \sqrt{5}$. [2]

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8 The curve C has parametric equations

$$x = t^2, \quad y = t - \tfrac{1}{3}t^3, \quad \text{for } 0 \leqslant t \leqslant 1.$$

Find

(i) the arc length of C, [5]

(ii) the surface area generated when C is rotated through 2π radians about the x-axis. [5]

9 The matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} -2 & 2 & 2 \\ 2 & 1 & 2 \\ -3 & -6 & -7 \end{pmatrix},$$

has an eigenvector $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. Find the corresponding eigenvalue. [2]

It is given that if the eigenvalues of a general 3×3 matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix},$$

are λ_1, λ_2 and λ_3 then

$$\lambda_1 + \lambda_2 + \lambda_3 = a + e + i$$

and

the determinant of $\mathbf{A}$ has the value $\lambda_1\lambda_2\lambda_3$.

Use these results to find the other two eigenvalues of the matrix $\mathbf{M}$, and find corresponding eigenvectors. [8]

10 It is given that $I_n = \displaystyle\int_0^{\frac{1}{4}\pi} \frac{\sin^{2n} x}{\cos x}\,\mathrm{d}x$, where $n \geqslant 0$. Show that

$$I_n - I_{n+1} = \frac{2^{-\left(n+\frac{1}{2}\right)}}{2n+1}. \qquad [5]$$

Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{\sin^6 x}{\cos x}\,\mathrm{d}x = \ln(1+\sqrt{2}) - \tfrac{73}{120}\sqrt{2}$. [5]

11 The line l_1 passes through the points $A(2, 3, -5)$ and $B(8, 7, -13)$. The line l_2 passes through the points $C(-2, 1, 8)$ and $D(3, -1, 4)$. Find the shortest distance between the lines l_1 and l_2. [5]

The plane Π_1 passes through the points A, B and D. The plane Π_2 passes though the points A, C and D. Find the acute angle between Π_1 and Π_2, giving your answer in degrees. [6]

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12 Answer only **one** of the following two alternatives.

EITHER

The curve C has parametric equations

$$x = t^2, \quad y = (2 - t)^{\frac{1}{2}}, \quad \text{for } 0 \leqslant t \leqslant 2.$$

Find

(i) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t, [5]

(ii) the mean value of y with respect to x over the interval $0 \leqslant x \leqslant 4$, [6]

(iii) the y-coordinate of the centroid of the region enclosed by C, the x-axis and the y-axis. [3]

OR

The curve C has equation

$$y = \frac{ax^2 + bx + c}{x + d},$$

where a, b, c and d are constants. The curve cuts the y-axis at $(0, -2)$ and has asymptotes $x = 2$ and $y = x + 1$.

(i) Write down the value of d. [1]

(ii) Determine the values of a, b and c. [6]

(iii) Show that, at all points on C, either $y \leqslant 3 - 2\sqrt{6}$ or $y \geqslant 3 + 2\sqrt{6}$. [7]

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