**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Level**

# MARK SCHEME for the May/June 2014 series

## 9231 FURTHER MATHEMATICS

**9231/21** Paper 2, maximum raw mark 100

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| Question Number | Mark Scheme Details | | Part Mark | Total |
|---|---|---|---|---|
| **1** | Equate impulse to momentum to find initial speed $v$ and Newton's law of restitution to find new speed: | $v = 4u,\ v' = ev = [-]\,3u$ M1 A1 | 2 | **2** |
| **2** | Find $v^2$ at both $A$ and $B$: | $v_A^{\,2} = \omega^2(a^2 - 0{\cdot}5^2)$ *and* $v_B^{\,2} = \omega^2(a^2 - 0{\cdot}75^2)$ B1 | | |
| | Find amplitude $a$ m from given K.E. ratio: | $\frac{1}{2} mv_A^{\,2} = (12/11)\ \frac{1}{2} mv_B^{\,2}$ | | |
| | | $11\,(a^2 - 0{\cdot}5^2) = 12\,(a^2 - 0{\cdot}75^2)$ | | |
| | | $a^2 = \frac{1}{4}(27 - 11) = 4,\ \ a = 2$ M1 A1 | 3 | |
| | Find $\omega$ from $v_{\max} = a\omega$: | $0{\cdot}6 = 2\omega,\ \ \omega = 0{\cdot}3$ B1 | | |
| | Find time ($\surd$ on $a$) at $A$ | $\omega^{-1}\sin^{-1}(0{\cdot}5/2)$ *or* $\omega^{-1}\cos^{-1}(0{\cdot}5/2)$ | | |
| | *or* at $B$, e.g.: | $\omega^{-1}\sin^{-1}(0{\cdot}75/2)$ *or* $\omega^{-1}\cos^{-1}(0{\cdot}75/2)$ M1 A1$\surd$ | | |
| | Combine correctly to find time from $A$ to $B$: | $\omega^{-1}\sin^{-1}(0{\cdot}75/2) - \omega^{-1}\sin^{-1}(0{\cdot}5/2)$ | | |
| | | *or* $\omega^{-1}\cos^{-1}(0{\cdot}5/2) - \omega^{-1}\cos^{-1}(0{\cdot}75/2)$ M1 | | |
| | Evaluate to 3 d.p.: | $= \omega^{-1}(0{\cdot}3844 - 0{\cdot}2527)$ or $\omega^{-1}(1{\cdot}318 - 1{\cdot}186)$ | | |
| | | $= 1{\cdot}2813 - 0{\cdot}8423$ | | |
| | | $4{\cdot}3937 - 3{\cdot}9547 = 0{\cdot}439$ [s] A1 | 5 | **8** |

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| **3** | Use conservation of momentum, e.g.: | $mv_A + 9mv_B = mu$ | M1 | | |
| | Use Newton's law of restitution (consistent signs): | $v_B - v_A = eu$ | M1 | | |
| | Relate $v_A$ to $v_B$ using K.E. (A.E.F.): | $\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{4}mu^2$ | M1 | | |
| | Combine two eqns to find $v_A$ and $v_B$ e.g.: | $v_A = (1 - 9e)u/10$, $v_B = (1 + e)u/10$ | | | |
| | | or $v_A$ , $v_B = -u/2$, u/6 [or $7u/10$, $u/30$] | M1 A1 | | |
| | Use in 3rd eqn to find $e$, e.g.: | $(1 - 9e)^2 + 9(1 + e)^2 = 50$ | | | |
| | (A0 if finally $\pm\frac{2}{3}$) | $90e^2 = 40$, $e = \frac{2}{3}$ | M1 A1 | 7 | |
| | Use Newton's law of restitution with | $v_C = 2v_B'$ , e.g.: $v_C - v_B' = ev_B$ , $v_B' = \frac{2}{3}v_B$ | B1 | | |
| | | [$v_B = u/6$, $v_B = u/9$, $v_C = 2u/9$] | | | |
| | Use conservation of momentum to find k: | $9mv_B' + kmv_C = 9mv_B$ | | | |
| | | $9v_B' + 2kv_B' = 13{\cdot}5v_B'$, $k = 9/4$ | M1 A1 | 3 | **10** |

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| **4** | **(i)** | Use conservation of energy at lowest point:<br>Use $F = ma$ radially at lowest point:<br>Eliminate $v^2$ to find $R$ $[v^2 = 2{\cdot}3\,ga]$: | $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$<br>$R - mg = mv^2/a$<br>$R = mu^2/a + 3mg = 3{\cdot}3\,mg$ | B1<br>B1<br>B1 | 3 | |
| | **(ii)** | Use conservation of energy at $B$ to find $V_B$ :<br>(A.E.F.) | $\frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga\sin\theta$<br>$V_B^2 = (0{\cdot}3 + 0{\cdot}5)ga$, $V_B = \sqrt{(0{\cdot}8\,ga)}$<br>*or* $2\sqrt{(ga/5)}$ *or* $0{\cdot}894\sqrt{(ga)}$ | M1A1<br><br>A1 | 3 | |
| | **(iii)** | Use vertical component $v_B$ of speed $V_B$ at $B$:<br>Find height $h$ reached above $B$:<br>Find height $h$ reached above level of $O$: | $v_B = V_B\cos\theta$ $[= \frac{1}{4}\sqrt{15}\,V_B = \sqrt{(\frac{3}{4}ga)}]$<br>$h = v_B^2/2g = 3a/8$<br>$h - a\sin\theta = 3a/8 - \frac{1}{4}a = a/8$ **A.G.** | M1<br>M1 A1<br>A1 | 4 | **10** |
| **5** | | Find MI of components about $A$:<br>(M1 for $BC$ or $CD$)<br><br>Find total MI about $A$:<br>(OR can first find total MI about centre of mass)<br>State or imply total mass acts at mid-point of $AC$<br><br>Use eqn of circular motion to find $d^2\theta/dt^2$ :<br>Approximate $\sin\theta$ by $\theta$ and substitute for $I$:<br><br>Find period $T = 2\pi/\omega$ with $\omega = \sqrt{(49\,g/384\,a)}$: | Glass $(3M/5)\{\frac{1}{3}(5a)^2 + 25a^2\} = 20\,Ma^2$<br>$AB$ $M\{\frac{1}{3}(4a)^2 + (4a)^2\} = 64\,Ma^2/3$<br>$AD$ $\frac{1}{3}M\{\frac{1}{3}(3a)^2 + (3a)^2\} = 4\,Ma^2$<br>$BC$ $\frac{1}{3}M\{\frac{1}{3}(3a)^2 + 73a^2\} = 76\,Ma^{2/3}$<br>$CD$ $M\{\frac{1}{3}(4a)^2 + 52a^2\} = 172\,Ma^{2/3}$<br><br>$I = 128\,Ma^2$ A.G.<br><br>$I\,d^2\theta/dt^2 = [-]\,(49Mg/15)\,5a\sin\theta$<br>$d^2\theta/dt^2 = -(49g/384a)\,\theta$<br><br>$T = 2\pi\sqrt{(384a/49g)}$<br>*or* $(16\pi/7)\sqrt{(6a/g)}$<br>*or* $17{\cdot}6\sqrt{(a/g)}$ (A.E.F.) | M1 A1<br>B1<br>B1<br>M1 A1<br>A1<br><br>A1<br><br>M1<br><br>M1 A1<br>A1<br><br>B1 | 8<br><br>5 | **13** |

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| **6** | | State or find the expected value of $X$: using $p = ¼$: | $E(X) = 1/p = 1/¼ = 4$ B1 | 1 | |
| | **(i)** | Find $P(X = 4)$: | $P(X = 4) = (¾)^3 ¼ = 27/256$ or 0·105 M1 A1 | 2 | |
| | **(ii)** | Find $P(X<6)$: | $P(X < 6) = 1 – (¾)^5$<br>or $\{1 + ¾ + (¾)^2 + (¾)^3 + (¾)^4\}¼$ | | |
| | | S.R. Using $p = ½$ can earn B0 M1 A0 M0 A0 | = 781/1024 or 0·763 M1 A1 | 2 | **5** |
| **7** | **(i)** | State probability density function of $T$: | $f(t) = 0·001 \exp(-0·001t)$ $(t \geq 0)$ B1<br>[ = 0 (otherwise *or* $t < 0$)] | 1 | |
| | **(ii)** | Find $P(T > 2000)$:<br>**S.R.** $1 – e^{-2} = 0·865$ earns B1 only (max 1/3)<br>State inequality for $t$ (lose A1 if = or ≤):<br>Solve for $t_{max}$:<br>(Omitting power 10 earns 0/4;<br>using $1 – (\exp(-0·001t))^{10}$ can earn M1 A0 M1 A0 only) | $P(t > 2000) = 1 – F(2000)$ M1<br>$= 1 – (1 – e^{-2}) = e^{-2}$ *or* 0·135 M1 A1<br>$(\exp(-0·001t))^{10} \geq$ [*or* >] 0·9 M1 A1<br>$t_{max} = (\ln 0·9)/(-0·01) = 10·5$ M1 A1 | 3<br>4 | **8** |

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| **8** | State hypotheses (B0 for $\bar{\chi}$ …): | $H_0: \mu_X = \mu_Y$, $H_1: \mu_X \neq \mu_Y$ B1 | | |
| | Estimate both popln. variances using two samples: (allow use of biased: $\sigma_{X,60}^2 = 236$ *or* $15{\cdot}36^2$) | $S_x^2 = (626\,220 - 6060^2/60) / 59$ [= 240 *or* $15{\cdot}49^2$] | | |
| | (allow use of biased: $\sigma_{Y,50}^2 = 265$ *or* $16{\cdot}28^2$) | *And* $s_Y^2 = (464\,500 - 4750^2/50) / 49$ [= 270·4 *or* $16{\cdot}44^2$] M1 A1 | | |
| | Estimate population variance for combined sample: | $s^2 = s_X^2/60 + s_Y^2/50$ | | |
| | (allow $\sigma_{X,60}^2/60 + \sigma_{Y,50}^2/50$: 9·233 *or* $3{\cdot}039^2$) | = 9·408 *or* $3{\cdot}067^2$ M1 A1 | | |
| | Calculate value of $z$ (to 2 d.p., either sign): | $z = (101 - 95) / s$ M1 | | |
| | | = 6/3·067 = 1·96 (*or* 1·97) A1 | | |
| | State or use correct tabular $z$ – value (to 2 d.p.): (or can compare 6 with e.g. 2·326 $s$ = 7·13 *or* 7·07) | $z_{0.99}$ = 2·326 *or* 2·33 (allow 2·36) B1 | | |
| | Correct conclusion (A.E.F, √on $z$ – values): | [Accept $H_0$] Claims are the same B1√ | | |
| | **S.R.** Assuming equal population variances: | Hypotheses; Explicit assumption (B1; B1) | | |
| | Find pooled estimate of common variance $s^2$: | : $s^2 = (626\,220 - 6060^2/60 +$ 464 500 – $4750^2/50) / 108$ (M1 A1) | | |
| | Calculate value of $z$ (to 2 d.p., either sign): | $z = 6 / s\sqrt{(1/60+1/50)} = 1{\cdot}97$ (M1 A1) | | |
| | | = 253·8 *or* $15{\cdot}93^2$ (A1) | | |
| | Tabular value; conclusion | As above (B1; B1√) | | **9** |

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| **9** | Find expected frequency $p$: | $p = 200 \int_2^3 (1 / x \ln 8)\, dx$<br>$= (200 / \ln 8) [\ln x]_2^3$<br>$= 200 \times 0{\cdot}1950 = 39{\cdot}00$ **A.G.** | M1A1 | | |
| | | $q = 21{\cdot}46$ *or* $21{\cdot}45$ | M1A1 | | |
| | Find $q$ by similar method *or* by using total of 200: | | | 4 | |
| | State (at least) null hypothesis: | $H_0$: f ($x$) fits data (A.E.F.) | B1 | | |
| | Calculate $\chi^2$ (to 3 s.f.): | $\chi^2 = 0{\cdot}202 + 0{\cdot}923 + 0{\cdot}678 + 0{\cdot}584$<br>$+ 1{\cdot}134 + 4{\cdot}134 + 3{\cdot}644 = 11{\cdot}3$ | M1A1 | | |
| | State or use correct tabular $\chi^2$ value (to 3 s.f.):: | $\chi^2_{6,\,0.95} = 12{\cdot}59$ | B1 | | |
| | Valid method for reaching conclusion: | Accept $H_0$ if $\chi^2 \leqslant$ tabular value | M1 | | |
| | Conclusion consistent with correct values (A.E.F): | Distribution fits observations | A1 | 6 | **10** |
| **10** | Find correlation coefficient $r$: | | | | |
| | $r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)(55\,991 - 639^2 / 10)\}}$ | | M1 A1 | | |
| | (A.E.F.; A0 if only 3 s.f. clearly used) | $= 18\,189{\cdot}6 / \sqrt{(46\,280{\cdot}4 \times 15\,158{\cdot}9)}$ | A1 | | |
| | | $= 0{\cdot}687$ | *A1 | 4 | |
| | State both hypotheses (B0 for $r$ …): | $H_0$: $\rho = 0$, $H_1$: $\rho \neq 0$ | B1 | | |
| | State or use correct tabular two-tail $r$-value: | $r_{10,\,5\%} = 0{\cdot}632$ | *B1 | | |
| | Valid method for reaching conclusion: | Reject $H_0$ if $\lvert r \rvert >$ tabular value | M1 | | |
| | Correct conclusion (A.E.F, dep *A1, *B1): | There is non-zero correlation | A1 | | |
| | Calculate gradient $p$ in $x - \bar{\chi} = p(y - \bar{\gamma})$: | $p = 18\,189{\cdot}6 / 15\,158{\cdot}9 = 1{\cdot}20$ | B1 | 4 | |
| | Find regression line of $x$ on $y$: | $x = 86{\cdot}6 + 1{\cdot}20 (y - 63{\cdot}9)$ | | | |
| | | $= 1{\cdot}20\, y + 9{\cdot}92$ | M1 A1 | 3 | **11** |

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| **11 A (i)** | Use Pythagoras to find $AB$: | $AB = \sqrt{(4a^2 + 12a^2)} = 4a$ | **A.G.** | M1 A1 | | |
| | Find $\angle SAB$: | $\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ *or* $\cos^{-1} 2a/4a$ | | | | |
| | | *or* $\tan^{-1} 2a\sqrt{3}/2a$ | | | | |
| **(ii)** | | $= 60°$ so $\angle SAB = 30°$ | **A.G.** | M1 A1 | 4 | |
| | *EITHER* | | | | | |
| | Resolve vertically and horizontally, e.g.: | $\frac{1}{2} N_A + \frac{1}{2}\sqrt{3}\, N_B + \frac{1}{2}\sqrt{3}\, F_A = W$ | | | | |
| | ($F_A$ may be in either direction) | *and* $\frac{1}{2}\sqrt{3}\, N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$ | | M1 A1 | | |
| | Eliminate $N_B + F_A$ to find $N_A$: | $N_A = \frac{1}{2} W$ | **A.G.** | A1 | | |
| | *OR* | | | | 3 | |
| **(iii)** | | | | | | |
| | Resolve in dirn. $PQ$ to find $N_A$: | $N_A = \frac{1}{2} W$ | **A.G.** | (M1 A1) | | |
| | Second resolution, e.g. in dirn. $PS$: | $N_B + F_A = \frac{1}{2}\sqrt{3}\, W$ | | (A1) | | |
| | Take moments, e.g. about $A$: | $\frac{1}{2}\sqrt{3}\, W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$ | | | | |
| | (A1 for each side of eqn) | $= N_B \times 2a$ | | M1 A1 A1 | | |
| | Solve to find $N_B$: | $N_B = \{(7\sqrt{3} - 6)/8\}\, W$ | | M1 A1 | | |
| | Use $N_B$ to find $F_A$: | $F_A = \sqrt{3}\, N_A - N_B$ *or* $\frac{1}{2}\sqrt{3}\, W - N_B$ | | | | |
| | | $= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.) | | M1 A1 | 7 | **14** |

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| **B** | Estimate population variance: | $s_P^2 = (236{\cdot}0 - 42{\cdot}8^2/8)/7$ | | | |
| | (allow biased here: $0{\cdot}8775$ or $0{\cdot}9367^2$) | $= 351/350$ or $1{\cdot}003$ or $1{\cdot}001^2$ | M1 | | |
| | Find confidence interval (allow $z$ in place of $t$) e.g.: | $42{\cdot}8/8 \pm t\sqrt{(s_P^2/8)}$ | M1 | | |
| | Use correct tabular $t$-value: | $t_{7,\,0.975} = 2{\cdot}365$ | A1 | | |
| | Evaluate C.I. correct to 2 d.p.: | $5{\cdot}35 \pm 0{\cdot}84$ or $[4{\cdot}51, 6{\cdot}19]$ | A1 | 4 | |
| | Formulate inequality for k (or equality for $k_{\max}$): | $(5{\cdot}35 - \text{k})/\sqrt{(s_P^2/8)} \geqslant$ [or $>$] $t$ | M1 | | |
| | Use correct tabular $t$-value: | $t_{7,\,0.9} = 1{\cdot}415$ | A1 | | |
| | Solve for $k_{\max}$ (A0 if = or $\leqslant$ was used for $k$ above): | $5{\cdot}35 - k \geqslant 0{\cdot}50$, $k_{\max} = 4{\cdot}85$ | A1 | 3 | |
| | State hypotheses (B0 for $\bar{x}$ …), e.g.: | $H_0$: $\mu_P = \mu_Q$, $H_1$: $\mu_P > \mu_Q$ | B1 | | |
| | State assumption (A.E.F.): | Normal distns. for [$P$ and] $Q$ | | | |
| | | *and* equal variances | B1 | | |
| | Estimate (pooled) common variance: | $s^2 = (7 \times 1{\cdot}003 + 11 \times 1{\cdot}962)/18$ | | | |
| | | $= 1{\cdot}589$ or $1{\cdot}261^2$ | M1 A1 | | |
| | Calculate value of $t$ (to 3 s.f.): | $t = (5{\cdot}35 - 4{\cdot}60)/(s\sqrt{(1/8 + 1/12)})$ | | | |
| | | $= 1{\cdot}30$ | M1 A1 | | |
| | Correct conclusion (A.E.F., $\surd$ on $t$): | $t < t_{18,\,0.9} = 1{\cdot}33$ so Q's mean is not less than P's | B1$\surd$ | 7 | **14** |

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