UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2013**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

2037772342

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

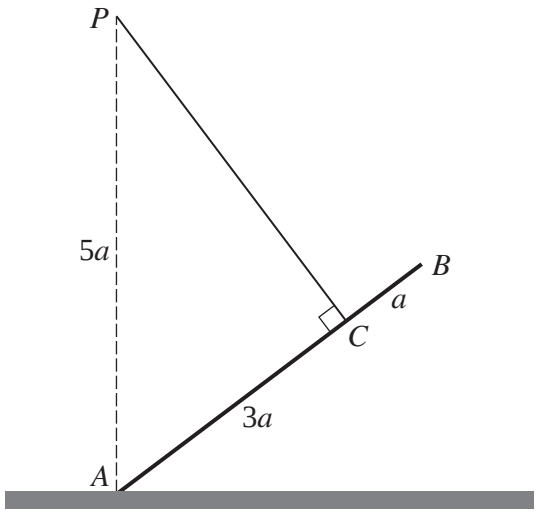
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

© UCLES 2013

1



A uniform rod AB, of mass m and length $4a$, rests with the end A on rough horizontal ground. The point C on AB is such that $AC = 3a$. A light inextensible string has one end attached to the point P which is at a distance $5a$ vertically above A, and the other end attached to C. The rod and the string are in the same vertical plane and the system is in equilibrium with angle ACP equal to 90° (see diagram). The coefficient of friction between the rod and the ground is μ. Show that the least possible value of μ is $\frac{24}{43}$. [8]

2 Three uniform small smooth spheres, A, B and C, have equal radii. Their masses are $4m$, $2m$ and m respectively. They lie in a straight line on a smooth horizontal surface with B between A and C. Initially A is moving towards B with speed u, B is at rest and C is moving in the same direction as A with speed $\frac{1}{2}u$. The coefficient of restitution between any two of the spheres is e. The first collision is between A and B. In this collision sphere A loses three-quarters of its kinetic energy. Show that $e = \frac{1}{2}$. [6]

Find the speed of B after its collision with C and deduce that there are no further collisions between the spheres. [5]

3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When P is hanging vertically below O, it is given a horizontal speed u. In the subsequent motion, P moves in a complete circle. When OP makes an angle θ with the downward vertical, the tension in the string is T. Show that

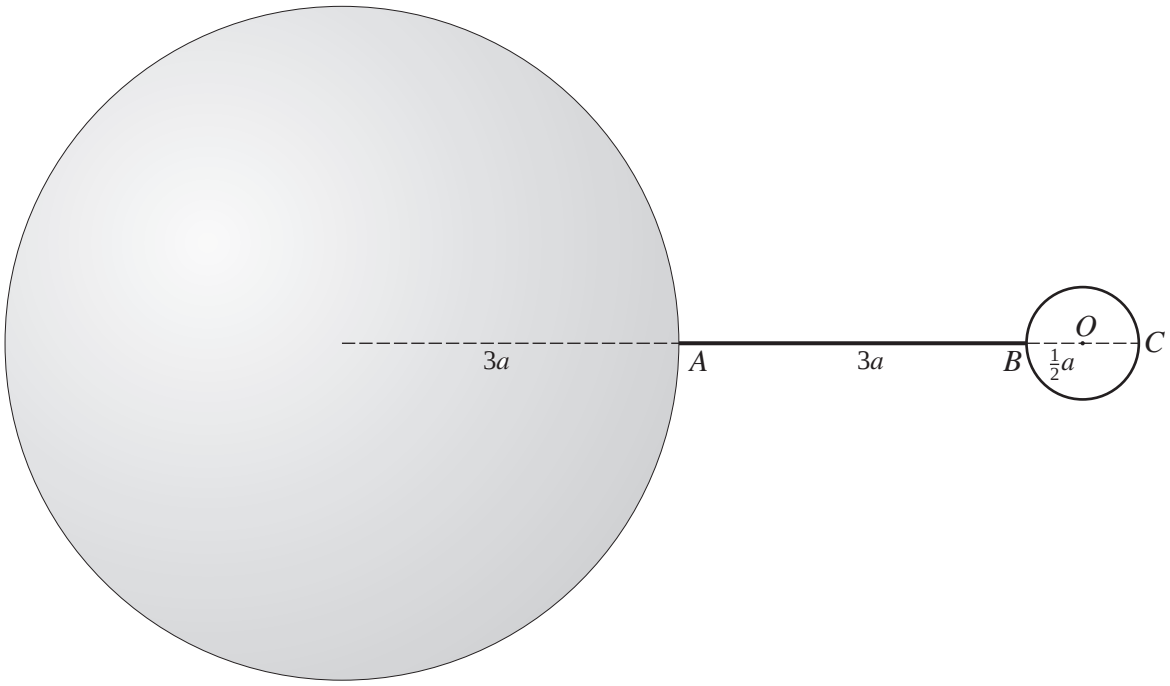
$$T = \frac{mu^2}{a} + mg(3\cos\theta - 2). \quad [5]$$

Given that the ratio of the maximum value of T to the minimum value of T is 3 : 1, find u in terms of a and g. [4]

Assuming this value of u, find the value of $\cos\theta$ when the tension is half of its maximum value. [3]

© UCLES 2013

4



The end A of a uniform rod AB, of mass $4m$ and length $3a$, is rigidly attached to a point on a uniform spherical shell, of mass λm and radius $3a$. The end B of the rod is rigidly attached to a point on a uniform ring. The ring has centre O, mass $4m$ and radius $\frac{1}{2}a$. The ring and the rod are in the same vertical plane. The line OBA, extended, passes through the centre of the spherical shell. BC is a diameter of the ring (see diagram). Show that the moment of inertia of this system, about a fixed horizontal axis through C perpendicular to the plane of the ring, is $(30 + 55\lambda)ma^2$. [6]

Given that the system performs small oscillations of period $2\pi\sqrt{\left(\frac{5a}{g}\right)}$ about this axis, find the value of λ. [6]

5 For a random sample of 12 observations of pairs of values (x, y), the product moment correlation coefficient is -0.456. Test, at the 5% significance level, whether there is evidence of negative correlation between the variables. [4]

6 The random variable X has distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 1 - \mathrm{e}^{-0.6x} & x \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

Identify the distribution of X and state its mean. [2]

Find

(i) $\mathrm{P}(X > 4)$, [2]

(ii) the median of X. [3]

7 A random sample of 80 observations of the continuous random variable X was taken and the values are summarised in the following table.

Interval	$2 \leqslant x < 3$	$3 \leqslant x < 4$	$4 \leqslant x < 5$	$5 \leqslant x < 6$
Observed frequency	36	29	9	6

It is required to test the goodness of fit of the distribution having probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{3}{x^2} & 2 \leqslant x < 6, \\ 0 & \text{otherwise.} \end{cases}$$

Show that the expected frequency for the interval $2 \leqslant x < 3$ is 40 and calculate the remaining expected frequencies. [4]

Carry out a goodness of fit test, at the 10% significance level. [5]

8 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{6}x & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

The random variable Y is defined by $Y = X^3$. Show that Y has probability density function g given by

$$\mathrm{g}(y) = \begin{cases} \frac{1}{18}y^{-\frac{1}{3}} & 8 \leqslant y \leqslant 64, \\ 0 & \text{otherwise.} \end{cases}$$ [6]

Find $\mathrm{E}(Y)$. [3]

9 A gardener P claims that a new type of fruit tree produces a higher annual mass of fruit than the type that he has previously grown. The old type of tree produced 5.2 kg of fruit per tree, on average. A random sample of 10 trees of the new type is chosen. The masses, x kg, of fruit produced are summarised as follows.

$$\Sigma x = 61.0 \qquad \Sigma x^2 = 384.0$$

Test, at the 5% significance level, whether gardener P's claim is justified, assuming a normal distribution. [6]

Another gardener Q has his own type of fruit tree. The masses, y kg, of fruit produced by a random sample of 10 trees grown by gardener Q are summarised as follows.

$$\Sigma y = 70.0 \qquad \Sigma y^2 = 500.6$$

Test, at the 5% significance level, whether the mean mass of fruit produced by gardener Q's trees is greater than the mean mass of fruit produced by gardener P's trees. You may assume that both distributions are normal and you should state any additional assumption. [8]

© UCLES 2013

10 Answer only **one** of the following two alternatives.

EITHER

A light elastic string has modulus of elasticity $\frac{3}{2}mg$ and natural length a. A particle of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A. Show that when the particle has fallen a distance ka from A, where $k > 1$, its kinetic energy is

$$\tfrac{1}{4}mga(10k - 3 - 3k^2).$$ [3]

Show that the particle first comes to instantaneous rest at the point B which is at a distance $3a$ vertically below A. [3]

Show that the time taken by the particle to travel from A to B is

$$\sqrt{\left(\frac{2a}{g}\right)} + \frac{2\pi}{3}\sqrt{\left(\frac{2a}{3g}\right)}.$$ [8]

OR

The regression line of y on x, obtained from a random sample of five pairs of values of x and y, has equation

$$y = x + k,$$

where k is a constant. The following table shows the data.

x	2	3	3	4	p
y	4	5	8	4	2

Find the two possible values of p. [8]

For the smaller of these two values of p, find

(i) the product moment correlation coefficient, [3]

(ii) the equation of the regression line of x on y. [3]

© UCLES 2013

BLANK PAGE

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.