UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

9498306157

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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1 A circular flywheel of radius 0.3 m, with moment of inertia about its axis 18 kg m^2, is rotating freely with angular speed 6 rad s^{-1}. A tangential force of constant magnitude 48 N is applied to the rim of the flywheel, in order to slow the flywheel down. Find the time taken for the angular speed of the flywheel to be reduced to 2 rad s^{-1}. [4]

2 Two particles, of masses $3m$ and m, are moving in the same straight line towards each other with speeds $2u$ and u respectively. When they collide, the impulse acting on each particle has magnitude $4mu$. Show that the total loss in kinetic energy is $\frac{4}{3}mu^2$. [6]

3 A particle P of mass m is projected horizontally with speed $\sqrt{\left(\frac{7}{2}ga\right)}$ from the lowest point of the inside of a fixed hollow smooth sphere of internal radius a and centre O. The angle between OP and the downward vertical at O is denoted by θ. Show that, as long as P remains in contact with the inner surface of the sphere, the magnitude of the reaction between the sphere and the particle is $\frac{3}{2}mg(1 + 2\cos\theta)$. [4]

Find the speed of P

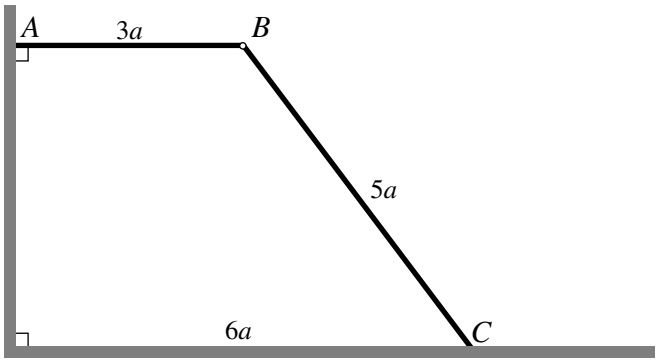
(i) when it loses contact with the sphere, [3]

(ii) when, in the subsequent motion, it passes through the horizontal plane containing O. (You may assume that this happens before P comes into contact with the sphere again.) [3]

4 AB is a diameter of a uniform circular disc D of mass $9m$, radius $3a$ and centre O. A lamina is formed by removing a circular disc, with centre O and radius a, from D. Show that the moment of inertia of the lamina, about a fixed horizontal axis l through A and perpendicular to the plane of the lamina, is $112ma^2$. [5]

A particle of mass $3m$ is now attached to the lamina at B. The system is free to rotate about the axis l. The system is held with B vertically above A and is then slightly displaced and released from rest. The greatest speed of B in the subsequent motion is $k\sqrt{(ga)}$. Find the value of k, correct to 3 significant figures. [7]

5



Two uniform rods AB and BC are smoothly jointed at B and rest in equilibrium with C on a rough horizontal floor and with A against a rough vertical wall. The rod AB is horizontal and the rods are in a vertical plane perpendicular to the wall. The rod AB has mass $3m$ and length $3a$, the rod BC has mass $5m$ and length $5a$, and C is at a distance $6a$ from the wall (see diagram). Show that the normal reaction exerted by the floor on the rod BC at C has magnitude $\frac{13}{2}mg$. [5]

The coefficient of friction at both A and C is μ. Find the least possible value of μ for which the rods do not slip at either A or C. [7]

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6 The probability that a particular type of light bulb is defective is 0.01. A large number of these bulbs are tested, one by one. Assuming independence, find the probability that the tenth bulb tested is the first to be found defective. [2]

The first defective bulb is the Nth to be tested. Write down the value of $\mathrm{E}(N)$. [1]

Find the least value of n such that $\mathrm{P}(N \leqslant n)$ is greater than 0.9. [3]

7 A random sample of 8 swimmers from a swimming club were timed over a distance of 100 metres, once in an outdoor pool and once in an indoor pool. Their times, in seconds, are given in the following table.

Swimmer	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Outdoor time	66.2	62.4	60.8	65.4	68.8	64.3	65.2	67.2
Indoor time	66.1	60.3	60.9	65.2	66.4	63.8	62.4	69.8

Assuming a normal distribution, test, at the 5% significance level, whether there is a non-zero difference between mean time in the outdoor pool and mean time in the indoor pool. [8]

8 The number of flaws in a randomly chosen 100 metre length of ribbon is modelled by a Poisson distribution with mean 1.6. The random variable X metres is the distance between two successive flaws. Show that the distribution function of X is given by

$$\mathrm{F}(x) = \begin{cases} 1 - \mathrm{e}^{-0.016x} & x \geqslant 0, \\ 0 & x < 0, \end{cases}$$

and deduce that X has a negative exponential distribution, stating its mean. [4]

Find

(i) the median distance between two successive flaws, [3]

(ii) the probability that there is a distance of at least 50 metres between two successive flaws. [2]

9 A random sample of 8 observations of a normal random variable X gave the following summarised data, where $\bar{x}$ denotes the sample mean.

$$\Sigma x = 42.5 \qquad \Sigma(x - \bar{x})^2 = 15.519$$

Test, at the 5% significance level, whether the population mean of X is greater than 4.5. [7]

Calculate a 95% confidence interval for the population mean of X. [3]

10 Random samples of employees are taken from two companies, A and B. Each employee is asked which of three types of coffee (Cappuccino, Latte, Ground) they prefer. The results are shown in the following table.

	Cappuccino	Latte	Ground
Company *A*	60	52	32
Company *B*	35	40	31

Test, at the 5% significance level, whether coffee preferences of employees are independent of their company. [7]

Larger random samples, consisting of N times as many employees from each company, are taken. In each company, the proportions of employees preferring the three types of coffee remain unchanged. Find the least possible value of N that would lead to the conclusion, at the 1% significance level, that coffee preferences of employees are not independent of their company. [4]

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11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light elastic string of modulus of elasticity $4mg$ and natural length l. The other end of the string is attached to a fixed point O. The particle rests in equilibrium at the point E, vertically below O. The particle is pulled down a vertical distance $\frac{1}{8}l$ from E and released from rest. Show that the motion of P is simple harmonic with period $\pi\sqrt{\left(\frac{l}{g}\right)}$. [4]

At an instant when P is moving vertically downwards through E, the string is cut. When P has descended a further distance $\frac{7}{16}l$ under gravity, it strikes a fixed smooth plane which is inclined at 30° to the horizontal. The coefficient of restitution between P and the plane is $\frac{1}{3}$. Show that the speed of P immediately after the impact is $\frac{1}{4}\sqrt{(5gl)}$. [8]

OR

A new restaurant S has recently opened in a particular town. In order to investigate any effect of S on an existing restaurant R, the daily takings, x and y in thousands of dollars, at R and S respectively are recorded for a random sample of 8 days during a six-month period. The results are shown in the following table.

Day	1	2	3	4	5	6	7	8
x	1.2	1.4	0.9	1.1	0.8	1.0	0.6	1.5
y	0.3	0.4	0.6	0.6	0.25	0.75	0.6	0.35

(i) Calculate the product moment correlation coefficient for this sample. [4]

(ii) Stating your hypotheses, test, at the 2.5% significance level, whether there is negative correlation between daily takings at the two restaurants and comment on your result in the context of the question. [5]

Another sample is taken over N randomly chosen days and the product moment correlation coefficient is found to be -0.431. A test, at the 5% significance level, shows that there is evidence of negative correlation between daily takings in the two restaurants.

(iii) Find the range of possible values of N. [3]

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