UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/21**

Paper 2 **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

* 2 4 6 5 6 1 8 7 6 7 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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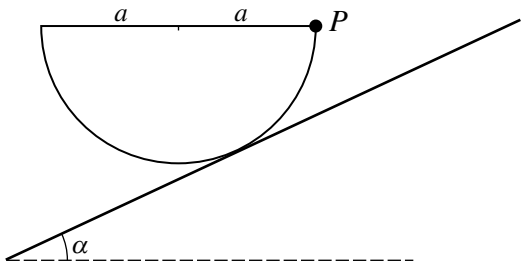
1 A particle oscillates in simple harmonic motion with centre O. When its distance from O is 3 m its speed is $16\,\mathrm{m\,s^{-1}}$, and when its distance from O is 4 m its speed is $12\,\mathrm{m\,s^{-1}}$. Find the period and amplitude of the motion. [5]

2 A particle of mass m is attached to the mid-point of a light elastic string. The string is stretched between two points A and B on a smooth horizontal surface, where $AB = 2a$. The string has modulus of elasticity λ and natural length $2l$, where $l < a$. The particle is in motion on the surface along a line passing through the mid-point of AB and perpendicular to AB. When the displacement of the particle from AB is x, the tension in the string is T. Given that x is small enough for x^2 to be neglected, show that

$$T = \frac{\lambda}{l}(a - l). \qquad [3]$$

The particle is slightly disturbed from its equilibrium position. Show that it will perform approximate simple harmonic motion and find the period of the motion. [4]

3



A uniform solid hemisphere, of radius a and mass M, is placed with its curved surface in contact with a rough plane that is inclined at an angle α to the horizontal. A particle P of mass m is attached to the rim of the hemisphere. The system rests in equilibrium with the rim of the hemisphere horizontal and P at the point on the rim that is closest to the inclined plane (see diagram). Given that the coefficient of friction between the plane and the hemisphere is $\frac{1}{2}$, show that

(i) $\tan\alpha \leqslant \frac{1}{2}$, [4]

(ii) $m \leqslant \dfrac{M(1+\sqrt{5})}{4}$. [5]

4 Two uniform spheres A and B, of equal radius, are at rest on a smooth horizontal table. Sphere A has mass $3m$ and sphere B has mass m. Sphere A is projected directly towards B, with speed u. The coefficient of restitution between the spheres is 0.6. Find the speeds of A and B after they collide. [5]

Sphere B now strikes a wall that is perpendicular to its path, rebounds and collides with A again. The coefficient of restitution between B and the wall is e. Given that the second collision between A and B brings A to rest, find e. [5]

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5 A particle P of mass m is placed at the point Q on the outer surface of a fixed smooth sphere with centre O and radius a. The acute angle between OQ and the upward vertical is α, where $\cos\alpha = \frac{9}{10}$. The particle is released from rest and begins to move in a vertical circle on the surface of the sphere. Show that P loses contact with the sphere when OP makes an angle θ with the upward vertical, where $\cos\theta = \frac{3}{5}$, and find the speed of P at this instant. [6]

Show that, in the subsequent motion, when P is at a distance $\frac{7}{5}a$ from the vertical diameter through O, its distance below the horizontal through O is $\frac{31}{30}a$. [6]

6 A random sample of residents in a town took part in a survey. They were asked whether they would prefer the local council to spend money on improving the local bus service or on improving the quality of road surfaces. The responses are shown in the following table, classified according to the area of the town in which the residents live.

	Area 1	Area 2	Area 3
Local bus service	73	36	30
Road surfaces	47	44	20

Using a 5% significance level, test whether there is an association between the area lived in and preference for improving the local bus service or improving the quality of road surfaces. [7]

7 A greengrocer claims that his cabbages have a mean mass of more than 1.2 kg. In order to check his claim, he weighs 10 cabbages, chosen at random from his stock. The masses, in kg, are as follows.

1.26 1.24 1.17 1.23 1.18 1.25 1.19 1.20 1.21 1.18

Stating any assumption that you make, test at the 10% significance level whether the greengrocer's claim is supported by this evidence. [8]

8 In a crossword competition the times, x minutes, taken by a random sample of 6 entrants to complete a crossword are summarised as follows.

$$\Sigma x = 210.9 \qquad \Sigma(x - \bar{x})^2 = 151.2$$

The time to complete a crossword has a normal distribution with mean μ minutes. Calculate a 95% confidence interval for μ. [5]

Assume now that the standard deviation of the population is known to be 5.6 minutes. Find the smallest sample size that would lead to a 95% confidence interval for μ of width at most 5 minutes. [4]

9 Mr Lee asserts that boys are slower than girls at completing a particular mathematical puzzle. In order to test his assertion, a random sample of 40 boys and a random sample of 60 girls are selected from a large group of students who attempted the puzzle. The times taken by the boys, b minutes, and the times taken by the girls, g minutes, are summarised as follows.

$$\Sigma b = 92.0 \quad \Sigma b^2 = 216.5 \quad \Sigma g = 129.8 \quad \Sigma g^2 = 288.8$$

Test at the 2.5% significance level whether this evidence supports Mr Lee's assertion. [9]

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10 The mid-day temperature, $x\,^{\circ}$C, and the amount of sunshine, y hours, were recorded at a winter holiday resort on each of 12 days, chosen at random during the winter season. The results are summarised as follows.

$$\Sigma x = 18.7 \quad \Sigma x^2 = 106.43 \quad \Sigma y = 34.7 \quad \Sigma y^2 = 133.43 \quad \Sigma xy = 92.01$$

(i) Find the product moment correlation coefficient for the data. [3]

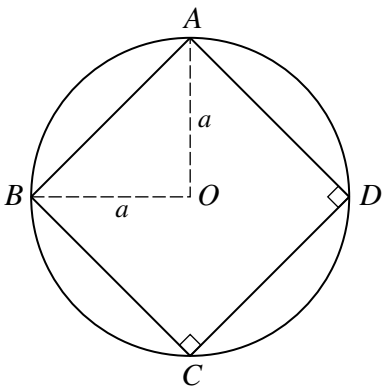
(ii) Stating your hypotheses, test at the 1% significance level whether there is a non-zero correlation between mid-day temperature and amount of sunshine. [4]

(iii) Use the equation of a suitable regression line to estimate the number of hours of sunshine on a day when the mid-day temperature is $2\,^{\circ}$C. [3]

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11 Answer only **one** of the following two alternatives.

EITHER

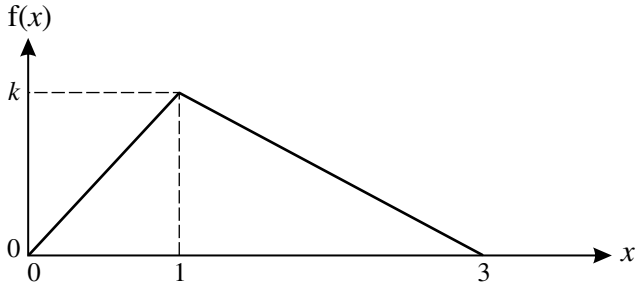


A rigid body is made from uniform wire of negligible thickness and is in the form of a square $ABCD$ of mass M enclosed within a circular ring of radius a and mass $2M$. The centres of the square and the circle coincide at O and the corners of the square are joined to the circle (see diagram). Show that the moment of inertia of the body about an axis through O, perpendicular to the plane of the body, is $\frac{8}{3}Ma^2$. [6]

Hence find the moment of inertia of the body about an axis l, through A, in the plane of the body and tangential to the circle. [2]

A particle P of mass M is now attached to the body at C. The system is able to rotate freely about the fixed axis l, which is horizontal. The system is released from rest with AC making an angle of 60° with the upward vertical. Find, in terms of a and g, the greatest speed of P in the subsequent motion. [6]

OR



The continuous random variable X takes values in the interval $0 \leqslant x \leqslant 3$ only. For $0 \leqslant x \leqslant 3$ the graph of its probability density function f consists of two straight line segments meeting at the point $(1, k)$, as shown in the diagram. Find k and hence show that the distribution function F is given by

$$\mathrm{F}(x) = \begin{cases} 0 & x \leqslant 0, \\ \frac{1}{3}x^2 & 0 < x \leqslant 1, \\ x - \frac{1}{2} - \frac{1}{6}x^2 & 1 < x \leqslant 3, \\ 1 & x > 3. \end{cases}$$ [6]

The random variable Y is given by $Y = X^2$. Find

(i) the probability density function of Y, [4]

(ii) the median value of Y. [4]

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